$T\bar{T}$ Deformation of Off-Shell Supersymmetry and Partially Broken Supersymmetry

Based on work in collaboration with *Prof.* **J.Yoon** [arXiv:2306.08030]

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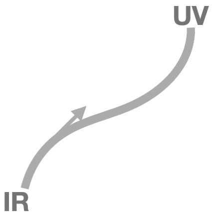
Korea Institute for Advanced Study (KIAS)

Overview

1. Special 2D irrelevant deformation: **$\mathbf{T\bar{T}}$ deformation**
2. Constructing SSB model: **Non-linear realization**
3. $T\bar{T}$**-deformed theories** and **non-linearly realized models**
4. Constructing $T\bar{T}$-deformed superaction
5. Remarks

Special 2D irrelevant deformation: $\mathrm{T\bar{T}}$ deformation

- **Irrelevant deformation** of 2D QFT which deform the theory into *non-local* theory.



- Deformed theory becomes non-local, but its **deformed energy spectrum** and **deformed S-matrix** can be tractable.
- Deformed theory is characterized by the $T\bar{T}$ **flow equation**:
  $\partial_\lambda S = \int d^2x \det(T_{\mu\nu})$

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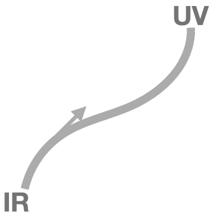

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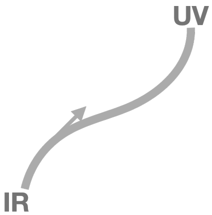

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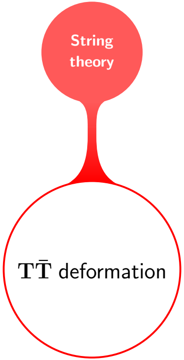
Have fun with $T\bar{T}$ deformation



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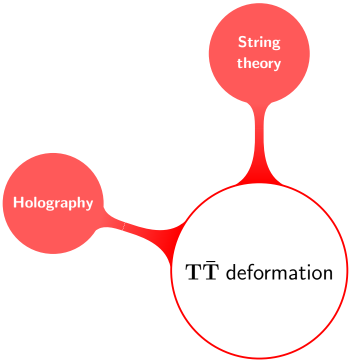

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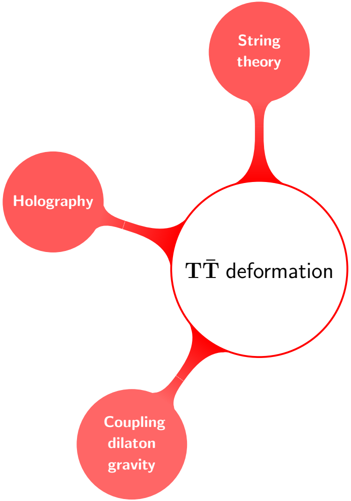

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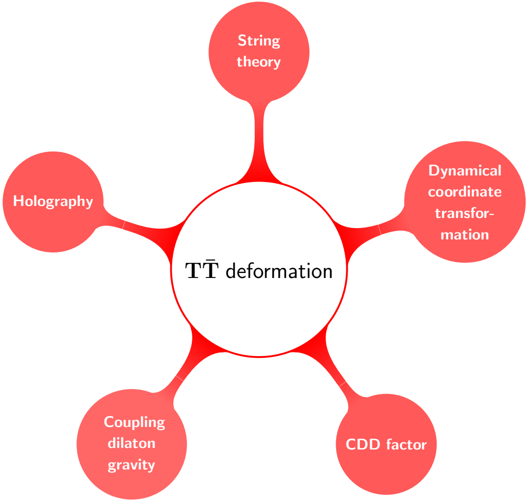

Example of $T\bar{T}$ deformed theories

Flow equation

$$\partial_\lambda S_\lambda = \int d^2x \;\det T^{(\lambda)}_{\mu\nu}$$

Seed theory $\xrightarrow{Flow\ eq.}$ **Deformed theory**

Free scalar theory $\longrightarrow$ Deformed scalar theory

Free fermion theory $\longrightarrow$ Deformed fermion theory

Free $\mathcal{N} = (1,1)$ theory $\longrightarrow$ Deformed $\mathcal{N} = (1,1)$ theory

Deformed free scalar theory

$$\mathcal{L}_\lambda = -\frac{1}{2\lambda}\left[\sqrt{1 - 8\lambda\partial_{++}\phi\partial_{=}\phi} - 1\right] \tag{1}$$

Deformed free fermion theory

$$\mathcal{L}_\lambda = i\psi_+\partial_=\psi_+ + i\psi_-\partial_{++}\psi_- + \lambda(\psi_+\partial_{++}\psi_+\psi_-\partial_=\psi_- - \psi_+\partial_=\psi_+\psi_-\partial_{++}\psi_-) \tag{2}$$

Deformed free $\mathcal{N} = (1,1)$ theory

$$\begin{aligned}\mathcal{L}_\lambda = & -\frac{1}{2\lambda}\left[\sqrt{1+2\chi} - 1\right] + \frac{1+\chi+\sqrt{1+2\chi}}{2\sqrt{1+2\chi}}(S_{++,=} + S_{=,++}) + \frac{2\lambda}{\sqrt{1+2\chi}}[(\partial_=\phi)^2 S_{++,++} + (\partial_{++}\phi)^2 S_{=,=}] \\ & + \lambda\frac{1+\chi-\chi^2+(1+2\chi)^{\frac{3}{2}}}{2(1+2\chi)^{\frac{3}{2}}} S_{++,++}S_{=,=} - \lambda\frac{1+3\chi+\chi^2+(1+2\chi)^{\frac{3}{2}}}{2(1+2\chi)^{\frac{3}{2}}} S_{++,=}S_{=,++} \\ & - \frac{2\lambda^2\chi}{(1+2\chi)^{\frac{3}{2}}}[(\partial_{++}\phi)^2 S_{++,=}S_{=,=} + (\partial_=\phi)^2 S_{++,++}S_{=,++}], \qquad \chi = -4\lambda\partial_{++}\phi\partial_=\phi\end{aligned} \tag{3}$$

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$T\bar{T}$ deformed Lagrangians

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Deformed free $\mathcal{N} = (1,1)$ theory

$$\mathcal{L}_\lambda = \text{Some complicated expression...}$$

Do $\mathcal{N} = (1,1)$ SUSY survive during the deformation?

Yes! $\Rightarrow$ $\exists$ **Supercharge** $Q^1_\pm = \int dx \; \psi_\pm \left(\pi \pm \phi'\right)$

SUSY survive under the $T\bar{T}$ deformation

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In fact there are more global charges $Q^2_\pm$ that form the 3D $\mathcal{N} = 2$ SUSY algebra(+toplogical charge) together with $Q^1_\pm$, but

$Q^2_\pm|0\rangle \neq 0$ $\Rightarrow$ **Spontaneously Symmetry Breaking**

What does **spontaneosuly symmetry breaking** have to do with $\mathrm{T\bar{T}}$ **deformation**?

Constructing SSB model: Non-linear realization

Spontaneously Symmetry Breaking

- When the **spontaneously symmetry breaking** (SSB) occurs, we get low-energy effective theory. To describe it, we do not need to know UV detail.

- Thus, there would be **a way to construct the effective theory based on SSB pattern**.

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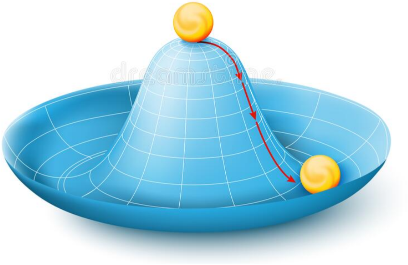
How can we effectively construct the SSB model?



“Non-linear realization”

a.k.a Coset construction, CCWZ formalism

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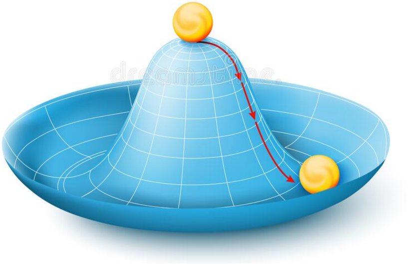

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Efficient way of constructing the symmetry broken model

- Constructing **SSB models**
- Constructing **string, brane and gravitational actions**

Internal symmetries: *Callan, Coleman, Wess and Zumino,*

Space-time symmetries: *Volkov and Ogievetsky.*

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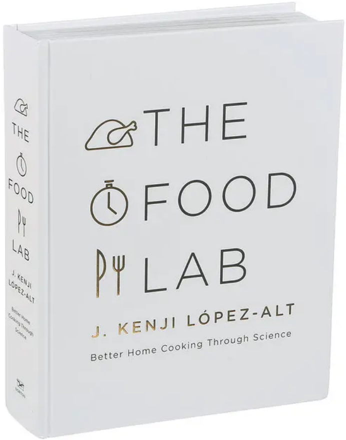
Cooking up the SSB model



1. Prepare the ingredients

   (Coset elements)

2. Follow the right recipe to cook

   (Maurer-Cartan form, Coset vielbein)

3. Bon Appétit!

   (Non-linear realization!)

Ingredients: Coset Representation

1. Let's start with the **symmetry breaking pattern** $\mathbf{G} \to \mathbf{H}$ and its **coset** $\mathbf{G}/\mathbf{H}$.

2. Lie algebra of G and H are $\mathfrak{g}$ and $\mathfrak{h}$. The generators of these algebras are $\{T_A, T_I\}$ and $\{T_I\}$.

3. Now we write the **exponential representation of coset element** as where we can always mod out the h. In hindsight, Z^A is **Goldstone field**.

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$$g(Z) = e^{Z^A T_A}$$

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Understanding the property of ingredients

Transformation of $\mathbf{Z}'$

$$g_0 g(Z) = g(Z')h(g_0, Z) \tag{4}$$

where we again mod out the $h(g_0, Z)$ to maintain the coset representation.

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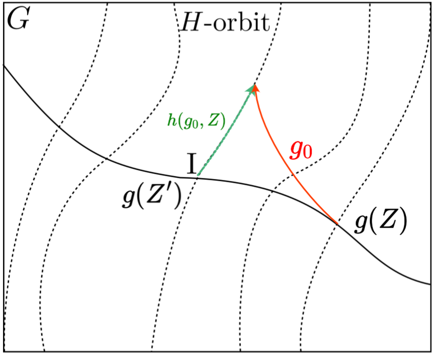

Building block to construct the invariant action

Maurer-Cartan form

$$A = g^{-1}dg = E^A T_A + E^I T_I \tag{5}$$

As $g(Z) \to g(Z') = g_0 g(Z) h^{-1}$, Cartan form A transform as

$$A \to A' \equiv E^A(Z')T_A + E^I(Z')T_I \tag{6}$$

where[1]

$$E^A(Z') = (Ad_h)^A_B E^B(Z) \tag{7}$$

[1] Assumption: $[\mathfrak{h},\mathfrak{h}] \subset \mathfrak{h}$, $[\mathfrak{h},\mathfrak{m}] \subset \mathfrak{m}$

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Coset vielbein and constructing invariant action

Coset vielbein

$$E^A(Z') = (Ad_h)^A_B E^B(Z) \tag{8}$$

Then we can construct the **invariant building block** by[2]

$$\int d^D x \ \det(E^A_\mu) = \int d^D x' \ \det(E'^A_\mu) \tag{9}$$

[2] For now, we are considering the spacetime symmetry breaking.

We can explicitly compute the **Maurer-Cartan form** from

$$g = e^{x^a P_a + \theta^\alpha Q_\alpha} \tag{10}$$

Then, we have

Maurer-Cartan form in VA model

$$A = g^{-1} dg = \underbrace{\left(\delta^a_\mu - i\bar{\theta}\gamma^a \partial_\mu \theta\right)}_{E^a_\mu} dx^\mu P_a + d\theta^\alpha Q_\alpha \tag{11}$$

Example: Volkov-Akulov action

From the Maurer-Cartan form A, we have

Coset zweibein

$$E_\mu^a = \delta_\mu^a - i\bar{\theta}\gamma^a\partial_\mu\theta \ . \quad (12)$$

By using this, we can construct the invariant action

Volkov-Akulov action

$$S_{VA} = -\frac{1}{\lambda}\int d^2x \ \det E_\mu^a \quad (13)$$

2D VA model and $T\bar{T}$ deformed fermion theory

After some rescaling, VA action can be expanded as

$$\begin{aligned}\mathcal{L}_{VA} &= -\frac{1}{\lambda}\int d^2x \ \det E^a_\mu \\ &= i\psi_+\partial_{=}\psi_+ + i\psi_-\partial_{+\!\!+}\psi_- + \lambda\big(\psi_+\partial_{+\!\!+}\psi_+\psi_-\partial_{=}\psi_- - \psi_+\partial_{=}\psi_+\psi_-\partial_{+\!\!+}\psi_-\big) - \tfrac{1}{\lambda}\end{aligned} \tag{14}$$

And recall that $T\bar{T}$ deformed free fermion theory

$$\mathcal{L}_\lambda = i\psi_+\partial_{=}\psi_+ + i\psi_-\partial_{+\!\!+}\psi_- + \lambda\big(\psi_+\partial_{+\!\!+}\psi_+\psi_-\partial_{=}\psi_- - \psi_+\partial_{=}\psi_+\psi_-\partial_{+\!\!+}\psi_-\big) \tag{15}$$

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$\mathrm{T\bar{T}}$ deformed fermion theory = Volkov-Akulov model

How about other $T\bar{T}$-deformed theories?
Can they also made from non-linear realization?

Deformed scalar theory?

Deformed fermion theory ✓

Deformed $\mathcal{N} = (1,1)$ theory?

$T\bar{T}$-deformed theories and non-linearly realized models

Deformed scalar theory via non-linear realization

Coset element

$$g = e^{x^a P_a + \phi(x) P_2 + \omega_{a2}(x) M^{a2}}$$

$$\Downarrow$$

Coset zweibein

$$A = g^{-1} dg = E^a{}_\mu P_a + \dots$$

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$$\mathcal{L} = -\frac{1}{2\lambda} \det\left(E^a{}_\mu\right) = \text{Deformed scalar theory}$$

Deformed $\mathcal{N} = (1,1)$ theory via non-linear realization

Coset element

$$g = e^{x^a P_a + \phi(x) P_2 + \omega_{a2}(x) M^{a2} + \theta^\alpha(x) Q_\alpha}$$

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Identification

How about other $T\bar{T}$-deformed theories?
Can they also made from non-linear realization?

Deformed scalar theory ✓

Deformed fermion theory ✓

Deformed $\mathcal{N} = (1,1)$ theory ✓

Constructing $T\bar{T}$-deformed superaction

Reverse-engineering to obtain deformed superaction

Free theory: $$S_0 = \int d^2x \, \mathcal{L}_0 = \int d^2x \, d^2\theta \, \frac{1}{2} \mathcal{D}_- \Phi \mathcal{D}_+ \Phi$$

Deformed theory: $$S_\lambda = \int d^2x \, \mathcal{L}_\lambda = \int d^2x \, d^2\theta \, \mathcal{A}_\lambda$$ (Deformed superaction)

Deformed free $\mathcal{N} = (1,1)$ theory: $\mathcal{L}_\lambda$

$$\mathcal{L}_\lambda = \begin{aligned} &-\frac{1}{2\lambda}\left[\sqrt{1+2\chi}-1\right] + \frac{1+\chi+\sqrt{1+2\chi}}{2\sqrt{1+2\chi}}(S_{\#,=}+S_{=,\#}) + \frac{2\lambda}{\sqrt{1+2\chi}}[(\partial_= \phi)^2 S_{\#,\#} + (\partial_\# \phi)^2 S_{=,=}] \\ &+\lambda\frac{1+\chi-\chi^2+(1+2\chi)^{\frac{3}{2}}}{2(1+2\chi)^{\frac{3}{2}}} S_{\#,\#} S_{=,=} - \lambda \frac{1+3\chi+\chi^2+(1+2\chi)^{\frac{3}{2}}}{2(1+2\chi)^{\frac{3}{2}}} S_{\#,=} S_{=,\#} \\ &-\frac{2\lambda^2\chi}{(1+2\chi)^{\frac{3}{2}}}[(\partial_\# \phi)^2 S_{\#,=} S_{=,=} + (\partial_= \phi)^2 S_{\#,\#} S_{=,\#}] \,, \qquad \chi \equiv -4\lambda \partial_\# \phi \partial_= \phi \end{aligned} \tag{16}$$

Superaction ansatz

Mass dimensions

$$[\mathcal{A}] = 1, \quad [\lambda] = -2, \quad [\Phi] = 0, \quad [\partial_{\pm\pm}] = 1, \quad [\mathcal{D}_{\pm}] = \frac{1}{2} \tag{17}$$

In addition to $\mathcal{D}_-\Phi\mathcal{D}_+\Phi$, there are 4 **Lorentz invariant** quadratic terms

$$A \equiv \partial_{++}\Phi\partial_{=}\Phi, \quad B \equiv \left(\mathcal{D}_-\mathcal{D}_+\Phi\right)^2, \quad C \equiv \mathcal{D}_+\Phi\partial_{=}\mathcal{D}_+\Phi, \quad D \equiv \mathcal{D}_-\Phi\partial_{++}\mathcal{D}_-\Phi$$

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Superaction ansatz

$$\mathcal{A}_\lambda = \Omega(\lambda A, \lambda B)\mathcal{D}_-\Phi\mathcal{D}_+\Phi$$

But... which superfield?

Vanilla scalar superfield

$$\Phi = \phi + i\theta^+\psi_+ + i\theta^-\psi_- + i\theta^+\theta^- F$$

Deformed scalar superfield

$$\Phi = \phi + i\theta^+\eta_+(\phi,\psi_\pm) + i\theta^-\eta_-(\phi,\psi_\pm) + i\theta^+\theta^- F$$

Deformed superaction and deformed superfield

Deformed superaction

$$\mathcal{A}_\lambda = \frac{\mathcal{D}_-\Phi\mathcal{D}_+\Phi}{1+\sqrt{1-2\lambda\mathcal{D}_-\mathcal{D}_+\left(\mathcal{D}_-\Phi\mathcal{D}_+\Phi\right)}} \tag{18}$$

Deformed superfield

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Off-shell broken supersymmetry

$$\delta_{\pm}^{\text{broken}}\Phi \sim \theta_{\pm} \pm i\lambda\mathcal{D}_{\pm}\Phi \qquad (21)$$

where $\hat{Q}_{\pm} \equiv -i\partial_{\pm} + 2\theta^{\pm}\partial_{\pm\pm}$, $\mathcal{D}_{\pm} \equiv -i\partial_{\pm} + 2\theta^{\pm}\partial_{\pm\pm}$.
These are compatible with the canonical results: $\delta\phi \sim \{Q_{\pm}, \phi\}_{Dirac}, \ldots$.

Another method for SUSY broken model: Constrained superfield method

By demanding the nilpotency of $\mathcal{N} = (2,2)$ scalar superfield ($\mathbf{\Phi}^2 = 0$), we have

$$S = \int d^2x \, d^2\theta \, \frac{\mathcal{D}_-\Phi\mathcal{D}_+\Phi}{1 + \sqrt{1 - 2\lambda\mathcal{D}_-\mathcal{D}_+ \left(\mathcal{D}_-\Phi\mathcal{D}_+\Phi\right)}} \tag{22}$$

which has the same form that we earned from the perturbative way.

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- We reviewed $\mathrm{T\bar{T}}$ **deformation** and **non-linear realization formalism**
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- Identified the result from **non-linear realization** and $\mathrm{T\bar{T}}$**-deformed theory**

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- We reviewed $\mathrm{T\bar{T}}$ **deformation** and **non-linear realization formalism**
- Explicit Example: $\mathrm{T\bar{T}}$ **-deformed fermion theory** and **Volkov-Akulov model**
- Identified the result from **non-linear realization** and $\mathrm{T\bar{T}}$**-deformed theory**
- Introduced construction of $\mathrm{T\bar{T}}$**-deformed superaction**

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- Genearl proof of the identification?
- Can this give us a hint about $\mathbf{T\bar{T}}$ **deformation in general dimensions**?
- Relation with non-linear σ-model or **other symmetry broken models**?
- What about $\mathbf{T\bar{T}}$ deformation of massive theories? Can they also described by the effective theories?

Thanks for your attention!