Abelian Decomposition and Two Types of Gluons in QCD

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Problems of QCD

- Proton is made of three quarks, without valence gluons. But obviously it contains gluons to bind them. If so, what is the binding gluons?
- Group theory tells that 2 of 8 gluons are color neutral. How can we separate the color neutral gluons?
- All non-Abelian gauge theories have the Abelian part. How can we separate the Abelian part gauge independently?

- ’tHooft conjectured the Abelian dominance to explain the color confinement. If so, what is the Abelian part of QCD and how do we separate this part?

- Nambu and Mandelstam proposed the monopole condensation for the confinement. How can we separate the monopole from the QCD potential?

- What is the essential difference between the Abelian QED and non-Abelian QCD that generates the color confinement and mass gap in QCD?

Millennium Problem

A. Abelian (“Cho”) decomposition: SU(2) QCD

- Let $(\hat{n}_1, \hat{n}_2, \hat{n}_3 = \hat{n})$ be an orthonormal basis and select the Abelian direction $\hat{n}$. Find the potential $\hat{A}_\mu$ which parallelizes $\hat{n}$,

$$D_\mu \hat{n} = \partial_\mu \hat{n} + g\vec{A}_\mu \times \hat{n} = 0,$$

$$\vec{A}_\mu \to \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = \tilde{A}_\mu + \tilde{C}_\mu,$$

$$\tilde{A}_\mu = A_\mu \hat{n}, \quad \tilde{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \vec{A}_\mu.$$

- $\hat{A}_\mu$ is Abelian but has a dual structure, made of the non-topological (Maxwellian) $\tilde{A}_\mu$ which describes the neutral binding gluon and the topological (Diracian) $\tilde{C}_\mu$ which describes the non-Abelian monopole.

- Obtain the gauge independent Abelian decomposition

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \quad (\hat{n}\cdot\vec{X}_\mu = 0).$$

$$\vec{F}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g\vec{X}_\mu \times \vec{X}_\nu,$$

$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g\hat{A}_\mu \times \hat{A}_\nu = (F_{\mu\nu} + H_{\mu\nu})\hat{n},$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu,$$

$$C_\mu = -\frac{1}{g}\hat{n}_1 \cdot \partial_\mu \hat{n}_2.$$

1. $\hat{A}_\mu$ describes the color neutral binding gluon (the neuron), but has the full SU(2) gauge degrees of freedom.
2. $\vec{X}_\mu$ describes the colored valence gluon (the chromon), but transforms gauge covariantly.

Two Types of Gluons!

Restricted QCD (RCD)

- Define RCD which describes the Abelian sub-dynamics with $\hat{A}_\mu$,

$$\mathcal{L}_{RCD} = -\frac{1}{4}\hat{F}_{\mu\nu}{}^2 = -\frac{1}{4}(F_{\mu\nu} + H_{\mu\nu})^2$$
$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2g}F_{\mu\nu}\hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n}) - \frac{1}{4g^2}(\partial_\mu\hat{n}\times\partial_\nu\hat{n})^2.$$

  It has the full SU(2) gauge symmetry yet is simpler than QCD, and has a dual structure with two potentials $\mathcal{A}_\mu$ and $\mathcal{C}_\mu$.

- $\hat{n}$ describes the monopole topology $\pi_2(S^2)$ and the vacuum topology $\pi_3(S^2)$.

"Non-Abelian" Dirac theory of monopole

B. Extended QCD (ECD)

- Adding the valence gluon we recover QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 = -\frac{1}{4}\hat{F}_{\mu\nu}^2$$

$$-\frac{1}{4}(\hat{D}_\mu\vec{X}_\nu - \hat{D}_\nu\vec{X}_\mu)^2 - \frac{g}{2}\hat{F}_{\mu\nu}\cdot(\vec{X}_\mu\times\vec{X}_\nu) - \frac{g^2}{4}(\vec{X}_\mu\times\vec{X}_\nu)^2.$$

1. QCD can be interpreted as RCD made of neuron which has the chromon as colored source.
2. This puts QCD to the background field formalism, with $\hat{A}_\mu$ and $\vec{X}_\mu$ as classical background and quantum fluctuation.
3. $\hat{n}$ describes the topological, not dynamical, degree.

- In this form QCD has two gauge symmetries, the classical (background) gauge symmetry

$$\delta\hat{A}_\mu = \frac{1}{g}\hat{D}_\mu\vec{\alpha}, \quad \delta\vec{X}_\mu = -\vec{\alpha}\times\vec{X}_\mu,$$

as well as the quantum (fast) gauge symmetry

$$\delta\hat{A}_\mu = \frac{1}{g}(\hat{n}\cdot D_\mu\vec{\alpha})\hat{n}, \quad \delta\vec{X}_\mu = \frac{1}{g}\hat{n}\times(D_\mu\vec{\alpha}\times\hat{n}).$$

- This justifies the existence of two types of gluons, the colorless neuron and the colored chromon, which play totally different roles.

New interpretation of QCD

Cho Decomposition

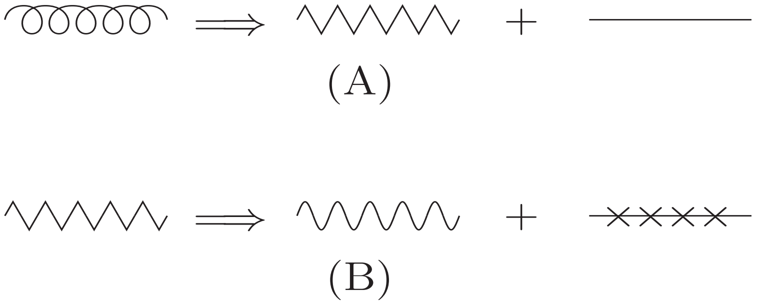

Figure: The gauge independent Abelian decomposition of QCD potential. (A) decomposes it to the restricted part and the chromon, and (B) decomposes the restricted part to the neuron and monopole.

Decomposition of QCD Feynman diagrams

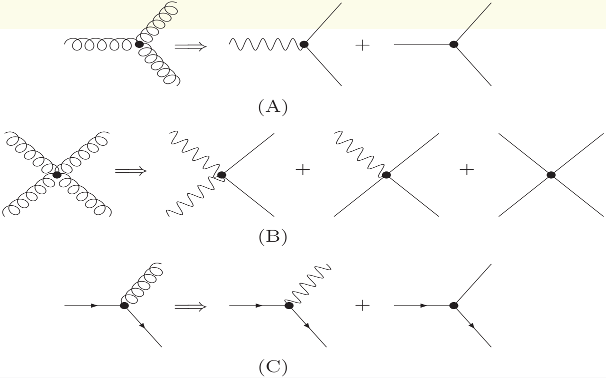

Figure: The Abelian decomposition of Feynman diagrams in SU(3) QCD. Notice that the monopole does not appear in the diagram because it describes a topological degree.

QCD Binding

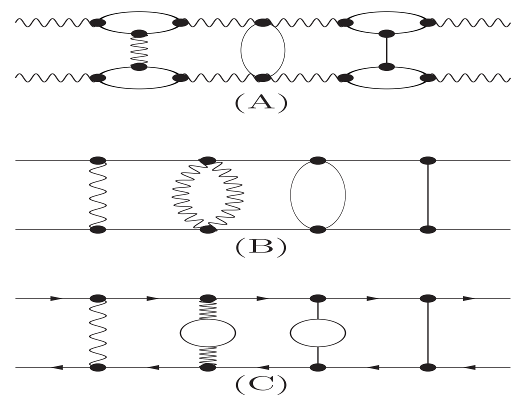

Figure: The possible Feynman diagrams of the neuron and chromon bindings. Two neuron binding is shown in (A), two chromon binding is shown in (B). In comparison the quark-antiquark binding is shown in (C).

Abelian Decomposition in Lattice QCD

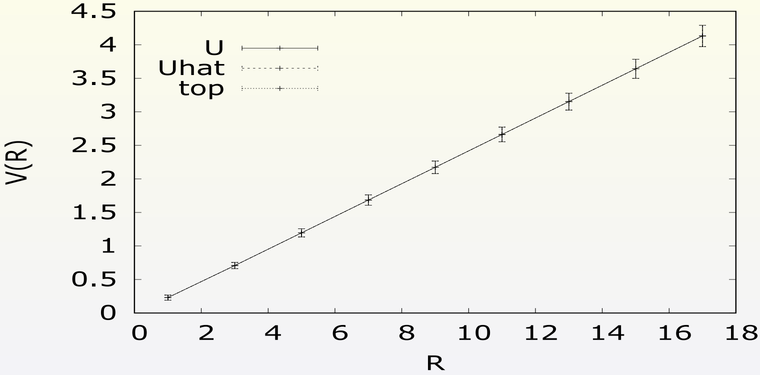

Figure: The Abelian dominance versus the monopole dominance in the lattice calculation. Here (U, Uhat, top) represent the full, Abelian, and monopole potentials.

A. Effective Action of SU(2) QCD

- Treating RCD as the classical part and adopting the gauge condition $\bar{D}_\mu \vec{X}_\mu = 0$ we have

$$\exp\big[iS_{eff}(\hat{A}_\mu)\big] \simeq \int \mathcal{D}\vec{X}_\mu \mathcal{D}\vec{X}_\mu^{(c)} \mathcal{D}\vec{c}\mathcal{D}\vec{c}^*$$

$$\exp\Big\{-i\int\big[\frac{1}{4}\hat{F}_{\mu\nu}^2 + \frac{1}{4}(\hat{D}_\mu\vec{X}_\nu - \hat{D}_\nu\vec{X}_\mu)^2 + \frac{g}{2}\hat{F}_{\mu\nu}\cdot(\vec{X}_\mu\times\vec{X}_\nu)$$

$$+\vec{c}^*\bar{D}_\mu D_\mu\vec{c} + \frac{1}{2\xi}(\bar{D}_\mu\vec{X}_\mu)^2\big]d^4x\Big\},$$

where $\vec{c}$ and $\vec{c}^*$ are the ghost fields.

- Choosing the monopole background $\hat{F}^{(b)}_{\mu\nu} = H\delta^1_{[\mu}\delta^2_{\nu]}\ \hat{n}$ and integrating out the chromon pair gauge invariantly, we have

$$V = \frac{H^2}{2}\Big[1 + \frac{11g^2}{24\pi^2}\big(\ln\frac{gH}{\mu^2} - c\big)\Big].$$

- Define the running coupling $\bar{g}$ by $\left.\frac{\partial^2 V}{\partial H^2}\right|_{H=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2}$ and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{24\pi^2}\big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{3}{2}\big), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial\bar{g}}{\partial\bar{\mu}} = -\frac{11\bar{g}^3}{24\pi^2}.$$

Monopole Condensation and Asymptotic Freedom

Effective potential of SU(2) QCD

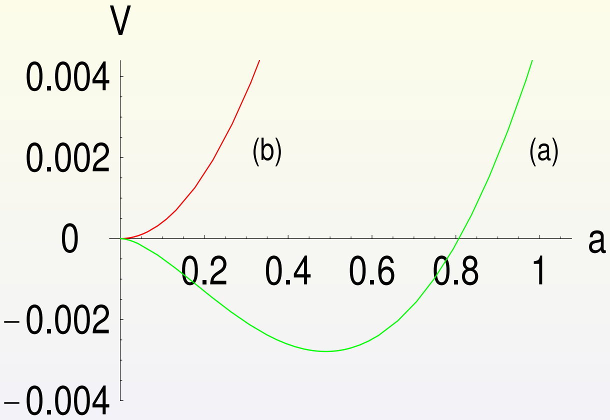

Figure: The one-loop effective potential of SU(2) QCD. Here (a) and (b) represent the effective potential and the classical potential.

Effective potential of SU(3) QCD

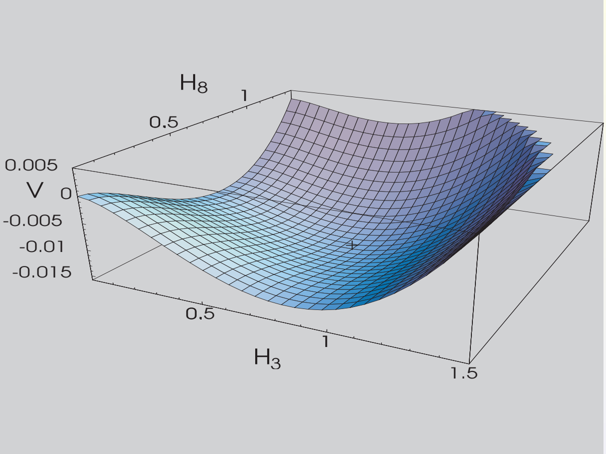

Figure: The effective potential with $\cos\theta = 0$, which has a unique minimum at $H = H' = H_0$ (or $H_1 = H_2 = H_3 = H_0$).

Observable Consequences

A. Two Types of Gluon Jets

- Experimental confirmation of the gluon jet and its separation from the quark jet has assured that QCD is the right theory of strong interaction.
- Jets are produced in two steps, parton shower and hadronization, and the jet shape is determined by the color factor.
- Quarks and gluons are known to have the color factor $C_F = 4/3$ and $C_A = 3$ ($C_A/C_F = 9/4$), so that the quark jet becomes sharper than the gluon jet.

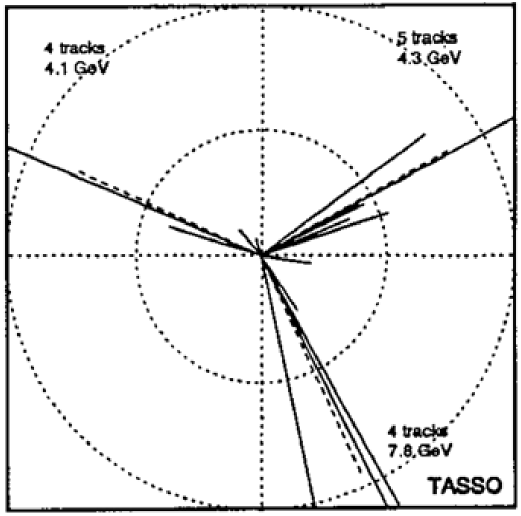

Figure: The 3 jets event of Tasso experiment which confirmed the existence of quark and gluon in QCD.

- But the Abelian decomposition tells that there are two types of gluons, the neuron and chromon. In fact, a simple number counting tells that one quarter of gluons are neurons and three quaters of gluons are chromons.

- The color factors of neurons and chromons are given by $C_n = 3/4$ and $C_c = 9/4$, so that $C_F : C_c : C_n = 4/3 : 9/4 : 3/4 \simeq 1.78 : 3 : 1$. This tells that the neuron jet should be sharpest.

- The parton shower (soft gluon radiation) of neuron, chromon, and quark are totally different. The has the leading order in neuron is $O(g^2)$, but in chromon and quark is $O(g)$.

Parton Shower in QCD Jets

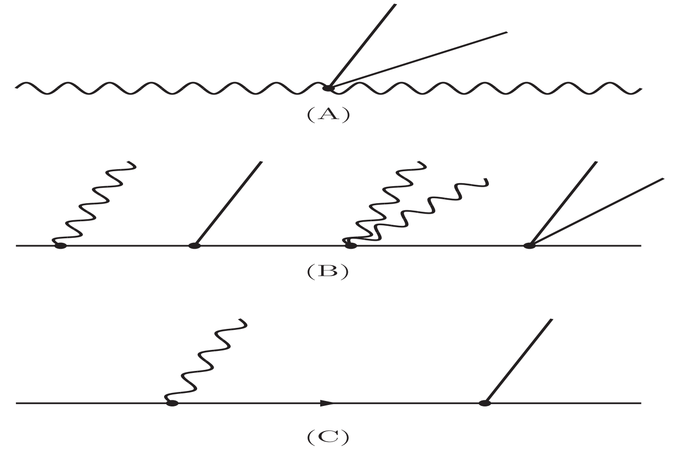

Figure: The parton shower of neuron, chromon, and quark in the first order Feynman diagram. The chromon and quark showers are of $O(g)$, and qualitatively similar, but the neuron shower is of $O(g^2)$ and qualitatively different.

- This predicts two types of gluon jets, the neuron jet and chromon jet. The neuron jet must have sharpest jet shape, least particle multiplicity, and ideal color dipole pattern. But the color factor and the parton shower pattern of the chromon strongly implies that the chromon jet most likely looks like the known gluon jet.

- If so, why have we not detect them?

1. Nobody looked for this, because there was no motivation to do that.
2. The experimental identification of gluon jet has been a very complicated process, with the success rate at best around 70%.

- Reanalyzing the existing gluon jet events at CERN we could confirm the existence of the neuron jet.

1. Consider the old ALEPH data on gluon jet coming from Z decay, and examine the sphericity and particle multiplicity of the gluon jet,

$$e^+e^- \to Z \to b\bar{b} + g.$$

2. Check if the gluon distribution has the following predicted shape shown in the figure.

No New Experiment!

Expected Gluon Jet Distribution

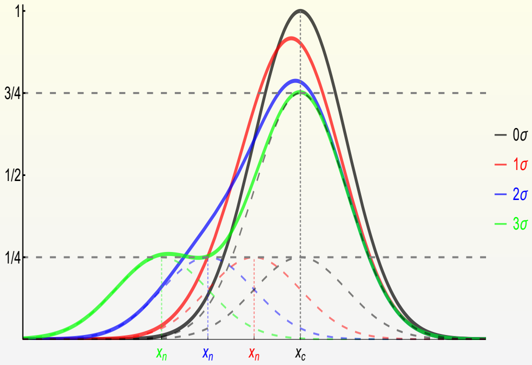

Figure: The expected gluon jet distribution against the jet shape (the sphericity) or the particle multiplicity. Here we have assmed that the distribution is Gaussian, and the red, blue, and green curves represent the gluon distribution when the distance between two peaks is 1,2, and 3 σ. The black curve represents when the neuron and chromon distributions are the same.

- Notice that the gluon distribution is asymmetric (i.e., tilted) against the peak axis, when the neuron and chromon jets are different.

- When the neuron peak is 3 σ and 2 σ apart, the left side is more populated by 64 % and 42 %. But when the distance becomes 1 σ, the left side is more populated only by 6 %.

- So, without trying to confirm the neuron jet, we could confirm the existence of two gluon jets confirming the asymmetry in the existing gluon jets.

B. Experimental Evidence

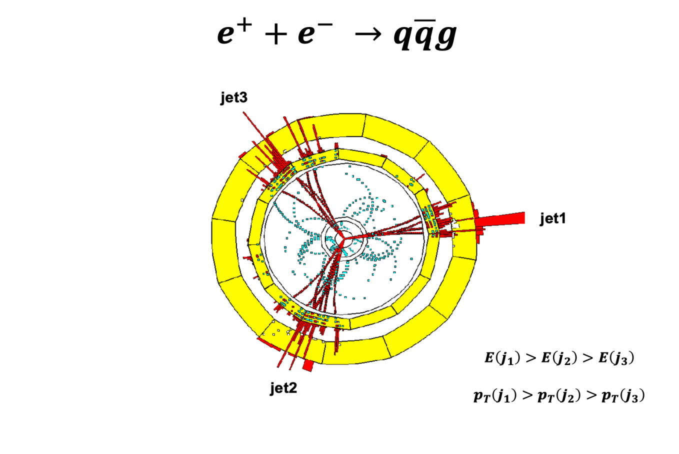

Figure: The $b\bar{b}g$ jets coming from the Z decay at ALEPH.

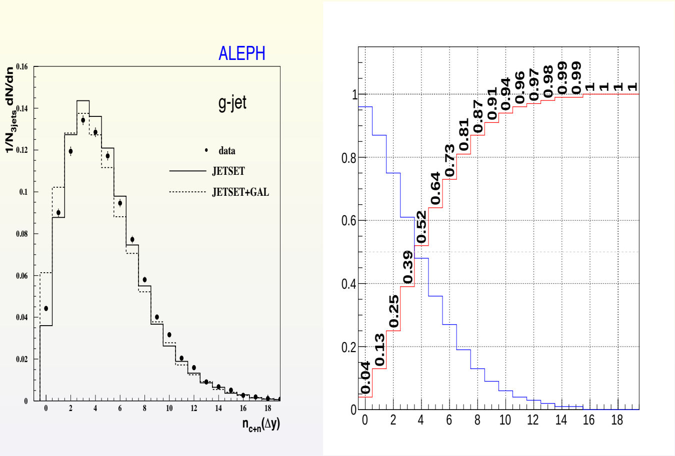

Figure: The asymmetry of the gluon distribution against the number of the particles at ALEPH.

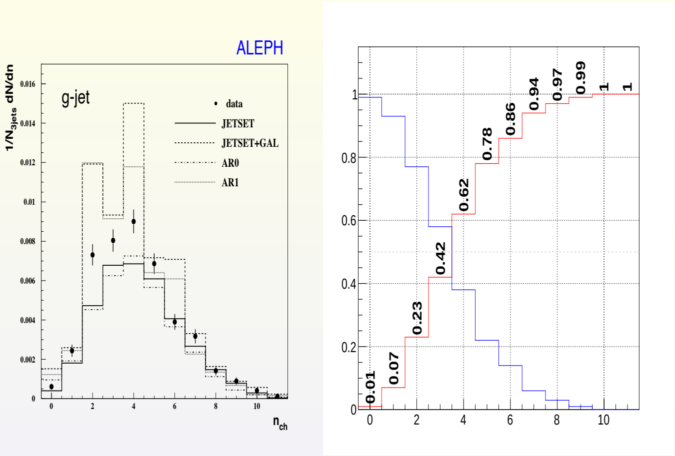

Figure: The asymmetry of the gluon distribution against the number of the charged particles at ALEPH.

- Notice that for the charged particle multiplicity, the left side is more populated by 20 %. This suggests the neuron peak 1.5 standard deviation away from the chromon peak. This clearly indicates the existence of the neuron jet.
- Moreover, the energy distribution of particles in the gluon jet of the ALEPH data clearly shows the excess of the particles in the high energy limit. This could be due to the neuron jet which is expected to produce more energetic particles.

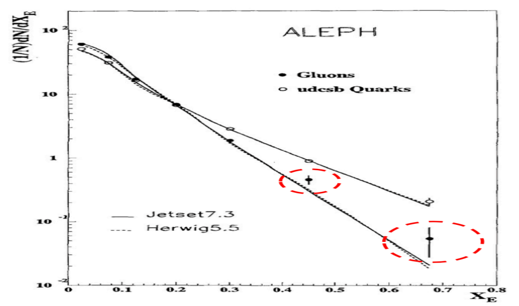

Figure: The energy distribution of the particles in the gluon jet in the ALEPH data which shows the excess of more energetic particles.

- Another unexpected place to look for the two types of gluons is the heavy ion collision experiment.
- In heavy ion collisions the gluon jets are expected to undergo the quenching when they pass through the quark gluon plasma of heavy ions which is absent in p-p collision. This should reduce the number of the gluon jets coming from the heavy ion collision.
- But this expected quenching has not been observed. This is because the quenching become negligible for the neuron jets, so that most of them come out unquenched.

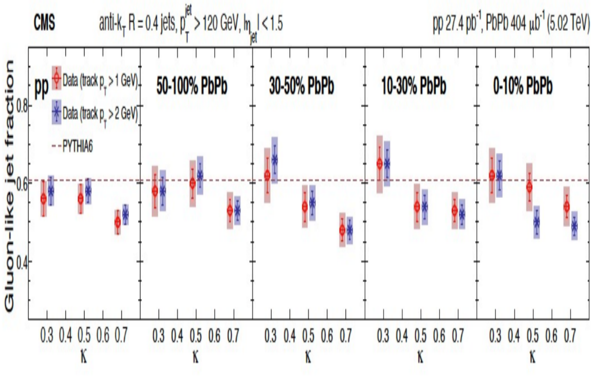

Figure: The CMS gluon jet data in Pb-Pb heavy ion collision compared to the gluon jets in p-p collision. The expected gluon quenching in Pb-Pb collision is hardly detectable.

C. Works Need To Be Done

- One could go further and check the sphericity (the shape) of the gluon jets, and see if the neuron jets are sharper than the chromon jets.
- Moreover, one should check if indeed the neuron jets have an ideal color dipole pattern, while the chromon jets have distorted dipole pattern.
- A best way to do this is to divide the gluon jets of the particle multiplicity to neuron and chromon jets, and check the jet shape and color dipole pattern of two groups.

- To show the existence of two types of gluons without any doubt, we have to know the jet shape, particle multiplicity, and color dipole pattern of the neuron and chromon jets separately.

- For this we need to implement the Abelian decomposition in Pythia and FastJet, and have the numerical predictions on the charactristic features of the neuron and chromon jets.

- This may not be simple, but doable.

Big Challenge Ahead!

Summary

- The Abelian decomposition is not just a mathematical proposition. It reveals the important hidden structures which simplifies the QCD dynamics greatly.

- It predicts two types of gluons, decomposes the Feynman diagram, simplifies the gauge symmetry, generalizes the quark model, and allows us to prove the monopole condensation.

- The existing ALEPH and CMS gluon jet data do have circumferential evidences of two types of gluons. This motivates us to analyse the gluon jets further to separate the neuron jets from the chromon jets.

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