Abelian Decomposition and Two Types of Gluon in QCD

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Motivation

Problems of QCD

- Proton is made of three quarks, without valence gluons. But obviously it contains gluons to bind them. If so, what is the binding gluons?
- Group theory tells that 2 of 8 gluons are color neutral. How can we separate the color neutral gluons?
- All non-Abelian gauge theories have the Abelian part. How can we separate the Abelian part gauge independently?

- 'tHooft conjectured the Abelian dominance to explain the color confinement. To prove this we need to know what is the Abelian part of QCD. How do we know this?
- Nambu and Mandelstam proposed the monopole condensation for the confinement. How can we separate the monopole from the QCD potential?
- What is the essential difference between the Abelian QED and non-Abelian QCD that generates the color confinement and mass gap in QCD?

Millennium Problem

A. Abelian ("Cho") decomposition of SU(2) QCD

- Let $(\hat{n}_1, \hat{n}_2, \hat{n}_3 = \hat{n})$ be an orthonormal basis and $\hat{n}$ be the Abelian direction. Impose the isometry to obtain the restricted potential $\hat{A}_\mu$,

$$D_\mu \hat{n} = \partial_\mu \hat{n} + g\vec{A}_\mu \times \hat{n} = 0,$$

$$\vec{A}_\mu \rightarrow \hat{A}_\mu = A_\mu \hat{n} - \frac{1}{g}\hat{n} \times \partial_\mu \hat{n} = \tilde{A}_\mu + \tilde{C}_\mu,$$

$$\tilde{A}_\mu = A_\mu \hat{n}, \quad \tilde{C}_\mu = -\frac{1}{g}\hat{n} \times \partial_\mu \hat{n}, \quad A_\mu = \hat{n} \cdot \vec{A}_\mu.$$

$\hat{A}_\mu$ is Abelian but has a dual structure, made of the non-topological (Maxwellian) $\tilde{A}_\mu$ which describes the neutral binding gluon and the topological (Diracian) $\tilde{C}_\mu$ which describes the non-Abelian monopole.

- Obtain the gauge independent Abelian decomposition

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu, \quad (\hat{n} \cdot \vec{X}_\mu = 0).$$
$$\vec{F}_{\mu\nu} = \hat{F}_{\mu\nu} + \hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu + g\vec{X}_\mu \times \vec{X}_\nu,$$
$$\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu + g\hat{A}_\mu \times \hat{A}_\nu = (F_{\mu\nu} + H_{\mu\nu})\hat{n},$$
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu,$$
$$C_\mu = -\frac{1}{g}\hat{n}_1 \cdot \partial_\mu \hat{n}_2.$$

1. $\hat{A}_\mu$ describes the color neutral binding gluon (the neuron), but has the full SU(2) gauge degrees of freedom.
2. $\vec{X}_\mu$ describes the colored valence gluon (the chromon), but transforms gauge covariantly.

Two Types of Gluons!

Restricted QCD (RCD)

- Define RCD which describes the Abelian sub-dynamics with $\hat{A}_\mu$,

$$\mathcal{L}_{RCD} = -\frac{1}{4}\hat{F}_{\mu\nu}{}^2 = -\frac{1}{4}(F_{\mu\nu} + H_{\mu\nu})^2$$
$$= -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2g}F_{\mu\nu}\hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n}) - \frac{1}{4g^2}(\partial_\mu\hat{n}\times\partial_\nu\hat{n})^2.$$

  It has the full SU(2) gauge symmetry yet is simpler than QCD, and has a dual structure with two potentials $\mathcal{A}_\mu$ and $\mathcal{C}_\mu$.

- $\hat{n}$ describes the monopole topology $\pi_2(S^2)$ and the vacuum topology $\pi_3(S^2)$.

"Non-Abelian" Dirac theory of monopole

Extended QCD (ECD)

- Adding the valence gluon we recover QCD

$$\mathcal{L}_{QCD} = -\frac{1}{4}\vec{F}_{\mu\nu}^2 = -\frac{1}{4}\hat{F}_{\mu\nu}^2$$

$$-\frac{1}{4}(\hat{D}_\mu\vec{X}_\nu - \hat{D}_\nu\vec{X}_\mu)^2 - \frac{g}{2}\hat{F}_{\mu\nu}\cdot(\vec{X}_\mu\times\vec{X}_\nu) - \frac{g^2}{4}(\vec{X}_\mu\times\vec{X}_\nu)^2.$$

1. QCD can be interpreted as RCD made of neuron which has the chromon as colored source.
2. This puts QCD to the background field formalism, with $\hat{A}_\mu$ and $\vec{X}_\mu$ as classical background and quantum fluctuation.
3. $\hat{n}$ describes the topological, not dynamical, degree.

- In this form QCD has two gauge symmetries, the classical (background) gauge symmetry

$$\delta \hat{A}_\mu = \frac{1}{g} \hat{D}_\mu \vec{\alpha}, \quad \delta \vec{X}_\mu = -\vec{\alpha} \times \vec{X}_\mu,$$

as well as the quantum (fast) gauge symmetry

$$\delta \hat{A}_\mu = \frac{1}{g} (\hat{n} \cdot D_\mu \vec{\alpha}) \hat{n}, \quad \delta \vec{X}_\mu = \frac{1}{g} \hat{n} \times (D_\mu \vec{\alpha} \times \hat{n}).$$

- This justifies us to call the colorless binding gluon the neuron and the colored valence gluon the chromon, and generaliizes the quark model to the quark and chromon model.

Abelianized QCD

- With $X_\mu = \frac{1}{\sqrt{2}}(X^1_\mu + iX^2_\mu)$ $(\vec{X}_\mu = X^1_\mu\ \hat{n}_1 + X^2_\mu\ \hat{n}_2)$, we can "abelianize" QCD gauge independently

$$\mathcal{L} = -\frac{1}{4}G^2_{\mu\nu} - \frac{1}{2}|\tilde{D}_\mu X_\nu - \tilde{D}_\nu X_\mu|^2 + igG_{\mu\nu}X^*_\mu X_\nu + \frac{g^2}{4}(X^*_\mu X_\nu - X^*_\nu X_\mu)^2,$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad B_\mu = A_\mu + C_\mu, \quad \tilde{D}_\mu = \partial_\mu + igB_\mu.$$

In this Abelianiztion the non-Abelian structure is not disappeared, but hidden.

B. SU(3) QCD

- Since the SU(3) QCD has two Abelian directions, the Abelian projection is given by two magnetic symmetries,

$$D_\mu \hat{n} = 0, \qquad D_\mu \hat{n}' = 0, \qquad (\hat{n}^2 = \hat{n}'^2 = 1)$$

where $\hat{n}$ and $\hat{n}' = \hat{n} * \hat{n}$ are λ_3-like and λ_8-like octet unit vectors.

- With this we have the following Abelian projection,

$$\vec{A}_\mu \to \hat{A}_\mu, \quad \hat{A}_\mu = A_\mu \hat{n} + A'_\mu \hat{n}' - \frac{1}{g}(\hat{n} \times \partial_\mu \hat{n} + \hat{n}' \times \partial_\mu \hat{n}')$$

$$A_\mu = \hat{n} \cdot \vec{A}_\mu, \quad A'_\mu = \hat{n}' \cdot \vec{A}_\mu, \quad \hat{n} \cdot \vec{X}_\mu = \hat{n}' \cdot \vec{X}_\mu = 0.$$

- $\hat{A}_\mu$ can be expressed by the three neurons of SU(2) subgroups in Weyl symmetric form

$$\hat{A}_\mu = \sum_p \frac{2}{3} \hat{A}^p_\mu, \quad (p = 1, 2, 3),$$

$$\hat{A}^p_\mu = A^p_\mu \ \hat{n}^p - \frac{1}{g} \hat{n}^p \times \partial_\mu \hat{n}^p = \tilde{A}^p_\mu + \tilde{C}^p_\mu,$$

$$A^1_\mu = A_\mu, \quad A^2_\mu = -\frac{1}{2} A_\mu + \frac{\sqrt{3}}{2} A'_\mu, \quad A^3_\mu = -\frac{1}{2} A_\mu - \frac{\sqrt{3}}{2} A'_\mu,$$

$$\hat{n}^1 = \hat{n}, \quad \hat{n}^2 = -\frac{1}{2} \hat{n} + \frac{\sqrt{3}}{2} \hat{n}', \quad \hat{n}^3 = -\frac{1}{2} \hat{n} - \frac{\sqrt{3}}{2} \hat{n}'.$$

- With this we have the Abelian decomposition of SU(3) QCD,

$$\vec{A}_\mu = \hat{A}_\mu + \vec{X}_\mu = \sum_p (\frac{2}{3}\hat{A}^p_\mu + \vec{W}^p_\mu), \quad \vec{X}_\mu = \sum_p \vec{W}^p_\mu,$$
$$\vec{W}^1_\mu = X^1_\mu \hat{n}_1 + X^2_\mu \hat{n}_2,\ \vec{W}^2_\mu = X^6_\mu \hat{n}_6 + X^7_\mu \hat{n}_7,\ \vec{W}^3_\mu = X^4_\mu \hat{n}_4 + X^5_\mu \hat{n}_5.$$

- $\vec{W}^p_\mu$ can be expressed by red, blue, and green chromons of SU(2) subgroups (R_μ, B_μ, G_μ),

$$R_\mu = \frac{X^1_\mu + iX^2_\mu}{\sqrt{2}},\ B_\mu = \frac{X^6_\mu + iX^7_\mu}{\sqrt{2}},\ G_\mu = \frac{X^4_\mu + iX^5_\mu}{\sqrt{2}}.$$

But unlike $\hat{A}^p_\mu$, they are mutually independent.

- From this we have the Weyl symmetric SU(3) RCD and QCD

$$\mathcal{L}_{RCD} = -\sum_p \frac{1}{6}(\hat{F}^p_{\mu\nu})^2,$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}\vec{F}^2_{\mu\nu} = -\sum_p \Big\{\frac{1}{6}(\hat{F}^p_{\mu\nu})^2 + \frac{1}{4}(\hat{D}^p_\mu \vec{W}^p_\nu - \hat{D}^p_\nu \vec{W}^p_\mu)^2$$
$$+\frac{g}{2}\hat{F}^p_{\mu\nu}\cdot(\vec{W}^p_\mu \times \vec{W}^p_\nu)\Big\} - \sum_{p,q} \frac{g^2}{4}(\vec{W}^p_\mu \times \vec{W}^q_\mu)^2$$
$$-\sum_{p,q,r} \frac{g}{2}(\hat{D}^p_\mu \vec{W}^p_\nu - \hat{D}^p_\nu \vec{W}^p_\mu)\cdot(\vec{W}^q_\mu \times \vec{W}^r_\mu)$$
$$-\sum_{p\neq q} \frac{g^2}{4}\big[(\vec{W}^p_\mu \times \vec{W}^q_\nu)\cdot(\vec{W}^q_\mu \times \vec{W}^p_\nu) + (\vec{W}^p_\mu \times \vec{W}^p_\nu)\cdot(\vec{W}^q_\mu \times \vec{W}^q_\nu)\big].$$

Cho Decomposition

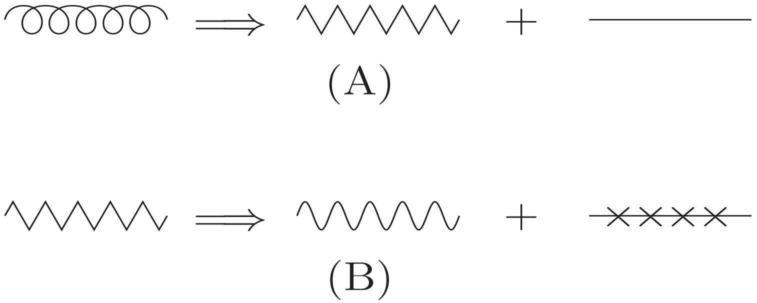

Figure: The gauge independent Abelian decomposition of QCD potential. (A) decomposes it to the restricted part and the chromon, and (B) decomposes the restricted part to the neuron and monopole.

Decomposition of QCD Feynman diagrams

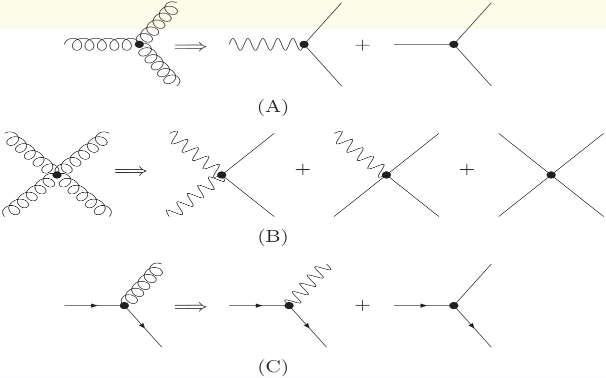

Figure: The Abelian decomposition of Feynman diagrams in SU(3) QCD. Notice that the monopole does not appear in the diagram because it describes a topological degree.

QCD Binding

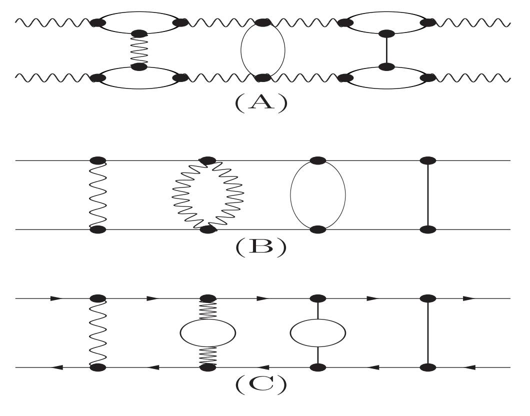

Figure: The possible Feynman diagrams of the neuron and chromon bindings. Two neuron binding is shown in (A), two chromon binding is shown in (B). In comparison the quark-antiquark binding is shown in (C).

- This shows that we can separate not only the Abelian potential but also the monopole potential gauge independently.

- This confirms that there are two types of gluon, the color neutral binding gluon and colored constituent gluon, which play totally different roles.

- Remaining problem: The decomposition of Feynman diagram necessitates a new Feynman rule for gluon interaction.

Big Challenge!!!

C. Abelian Dominance or Monopole Dominance

- With the Abelian decomposition we can prove the Abelian dominance, that the restricted potential generates the confining force in the Wilson loop integral.
- Moreover, implementing the Abelian decomposition on lattice, we can demonstrate the monopole dominance, that the monopole is responsible for the confinement.
- But this does not tell how the monopole confines the color.

Lattice QCD

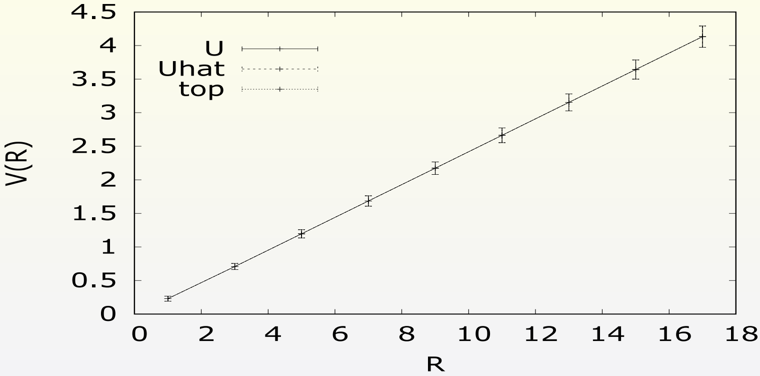

Figure: The Abelian dominance versus the monopole dominance in the lattice calculation. Here (U, Uhat, top) represent the full, Abelian, and monopole potentials.

A. Effective Action of SU(2) QCD

- Treating RCD as the classical part and adopting the gauge condition $\bar{D}_\mu \vec{X}_\mu = 0$ we have

$$\exp\left[iS_{eff}(\hat{A}_\mu)\right] \simeq \int \mathcal{D}\vec{X}_\mu \mathcal{D}\vec{X}_\mu^{(c)} \mathcal{D}\vec{c}\mathcal{D}\vec{c}^*$$

$$\exp\Big\{ - i\int [\frac{1}{4}\hat{F}_{\mu\nu}^2 + \frac{1}{4}(\hat{D}_\mu \vec{X}_\nu - \hat{D}_\nu \vec{X}_\mu)^2 + \frac{g}{2}\hat{F}_{\mu\nu} \cdot (\vec{X}_\mu \times \vec{X}_\nu)$$

$$+\vec{c}^* \bar{D}_\mu D_\mu \vec{c} + \frac{1}{2\xi}(\bar{D}_\mu \vec{X}_\mu)^2] d^4x \Big\},$$

where $\vec{c}$ and $\vec{c}^*$ are the ghost fields.

- Choosing the monopole background $\hat{F}^{(b)}_{\mu\nu} = H\delta^1_{[\mu}\delta^2_{\nu]}\ \hat{n}$ and integrating out the chromon pair gauge invariantly, we have

$$V = \frac{H^2}{2}\Big[1 + \frac{11g^2}{24\pi^2}\big(\ln\frac{gH}{\mu^2} - c\big)\Big].$$

- Define the running coupling $\bar{g}$ by $\frac{\partial^2 V}{\partial H^2}\Big|_{H=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2}$ and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{24\pi^2}\big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{3}{2}\big), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial\bar{g}}{\partial\bar{\mu}} = -\frac{11\bar{g}^3}{24\pi^2}.$$

Monopole Condensation and Asymptotic Freedom

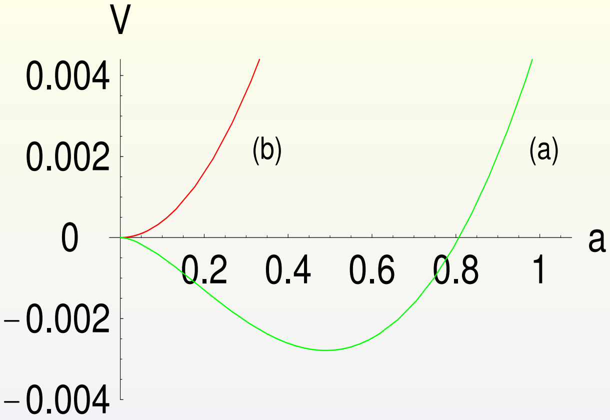

Figure: The one-loop effective potential of SU(2) QCD. Here (a) and (b) represent the effective potential and the classical potential.

- In general for arbitray constant monopole background $\bar{H}_{\mu\nu}$ we find

$$\mathcal{L}_{eff} = \begin{cases} -\dfrac{H^2}{2} - \dfrac{11g^2H^2}{48\pi^2}\Big(\ln\dfrac{gH}{\mu^2} - c\Big), & E = 0 \\ -\dfrac{E^2}{2} + \dfrac{11g^2E^2}{48\pi^2}\Big(\ln\dfrac{gE}{\mu^2} - c\Big) - i\dfrac{11g^2E^2}{96\pi}, & H = 0 \end{cases}$$

$$c = 1 - \ln 2 - \frac{24}{11}\zeta'(-1, \frac{3}{2}) = 0.94556...$$

- The negative imaginary part in the chromo-electric background tells that the chromo-electric field annihilates the chromon pairs. This is the origin of the asymptotic freedom.

B. SU(3) QCD

- Similarly we have the Weyl symmetric SU(3) QCD effective Lagrangian

$$\mathcal{L}_{eff} = \begin{cases} -\sum_p \Big(\frac{H_p^2}{3} + \frac{11g^2 H_p^2}{48\pi^2}(\ln \frac{gH_p}{\mu^2} - c)\Big), & (E_p = 0) \\ \sum_p \Big(\frac{E_p^2}{3} + \frac{11g^2 E_p^2}{48\pi^2}(\ln \frac{gE_p}{\mu^2} - c) \\ \qquad - i\frac{11g^2}{96\pi} E_p^2\Big). & (H_p = 0) \end{cases}$$

- This assures that the essential features of SU(2) QCD remains the same. In particular, this tells that the chromo-electric field makes the pair annihilation of chromon.

- The effective potential for the monopole background is given by

$$V = \frac{3}{4}\sum_p H_p^2 + \frac{11g^2}{48\pi^2}\sum_p H_p^2 \ln\big(\frac{gH_p}{\mu^2} - c\big).$$

- Define the renormalized coupling $\bar{g}$ by

$$\forall_p \quad \frac{\partial^2 V}{\partial H_p^2}\Big|_{H_1=H_2=H_3=\bar{\mu}^2} = \frac{g^2}{\bar{g}^2},$$

and find

$$\frac{1}{\bar{g}^2} = \frac{1}{g^2} + \frac{11}{16\pi^2}\big(\ln\frac{\bar{\mu}^2}{\mu^2} - c + \frac{5}{4}\big), \quad \beta(\bar{\mu}) = \bar{\mu}\frac{\partial\bar{g}}{\partial\bar{\mu}} = -\frac{11\bar{g}^3}{16\pi^2}.$$

Asymptotic Freedom!

Effective potential of SU(3) QCD

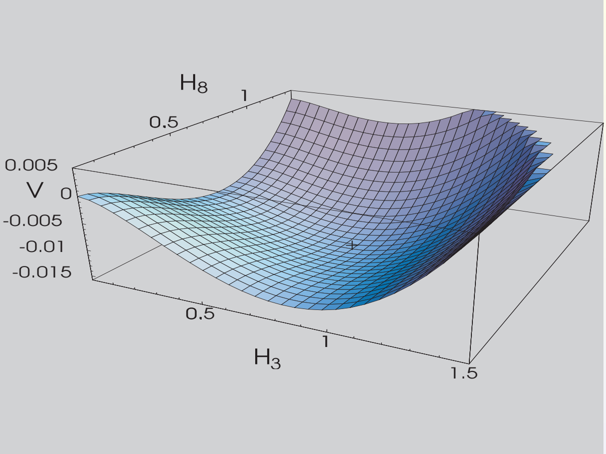

Figure: The effective potential with $\cos\theta = 0$, which has a unique minimum at $H = H' = H_0$ (or $H_1 = H_2 = H_3 = H_0$).

Observable Consequences

A. Two Types of Gluon Jets

- Experimental confirmation of the gluon jet and its separation from the quark jet has assured that QCD is the right theory of strong interaction.
- Jets are produced in two steps, parton shower and hadronization, and the jet shape is determined by the color factor.
- Quarks and gluons are known to have the color factor $C_F = 4/3$ and $C_A = 3$ $(C_A/C_F = 9/4)$, so that the quark jet becomes sharper than the gluon jet.

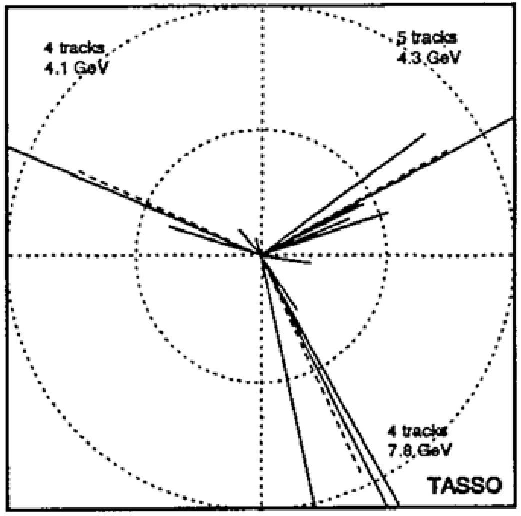

Figure: The 3 jets event of Tasso experiment which confirmed the existence of quark and gluon in QCD.

- The color factors of neurons and chromons are given by $C_n = 3/4$ and $C_c = 9/4$, so that $C_F : C_c : C_n = 4/3 : 9/4 : 3/4 \simeq 1.78 : 3 : 1$. This tells that the neuron jet should be sharpest.

- The parton shower (soft gluon radiation) of neuron, chromon, and quark are totally different. The has the leading order in neuron is $O(g^2)$, but in chromon and quark is $O(g)$.

- This predicts two types of gluon jets, the neuron jet and chromon jet which have different shapes, particle multiplicity, and color dipole pattern.

Parton Shower in QCD Jets

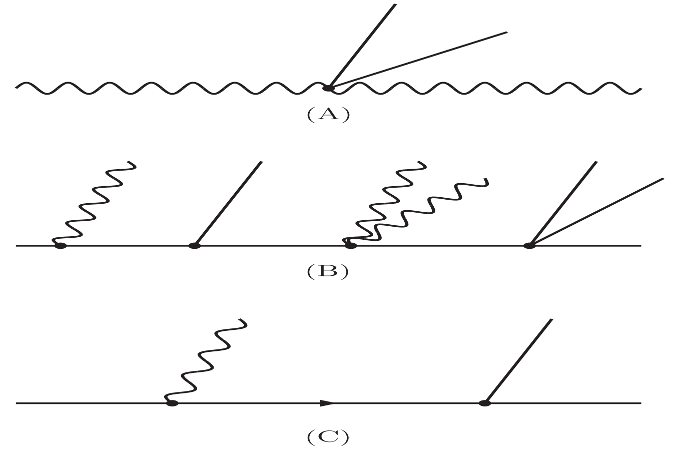

Figure: The parton shower of neuron, chromon, and quark in the first order Feynman diagram. The chromon and quark showers are of $O(g)$, and qualitatively similar, but the neuron shower is of $O(g^2)$ and qualitatively different.

- If so, why have we not detect them?

  1. Nobody looked for this, because there was no motivation to do that.
  2. The experimental identification of gluon jet has been a very complicated process, with the success rate at best around 70%.

- How can we detect them?

  1. The color factor and the parton shower pattern strongly implies that the known gluon jets could actually be the chromon jets.
  2. One quater of the gluon jet should actually be the neuron jet. It must have sharpest jet shape, least particle multiplicity, and ideal color dipole pattern.

- Reanalyzing the existing gluon jet events (with no new experiment) at LHC we could find the neuron jet.

  1. Consider the old ALEPH data on gluon jet coming from $b\bar{b}g$ three jet event, and examine the sphericity and the particle multiplicity of the gluon,

$$e^+e^- \to Z \to b\bar{b} + g.$$

  2. Check if the gluon distribution has the following predicted shape shown in the figure.

Expected Gluon Jet Distribution

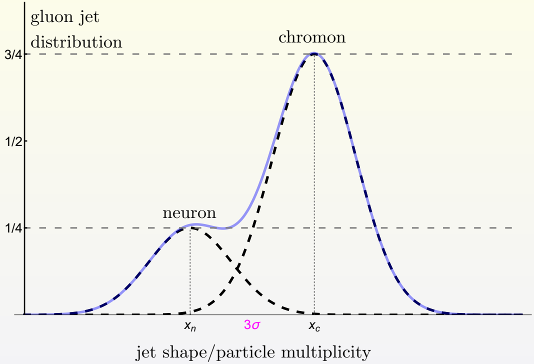

Figure: The expected gluon jet distribution against the jet shape (the sphericity) or the particle multiplicity. Here we have assmed that the distribution is Gaussian, and that the distance between two peaks is 3σ.

Expected Gluon Jet Distribution

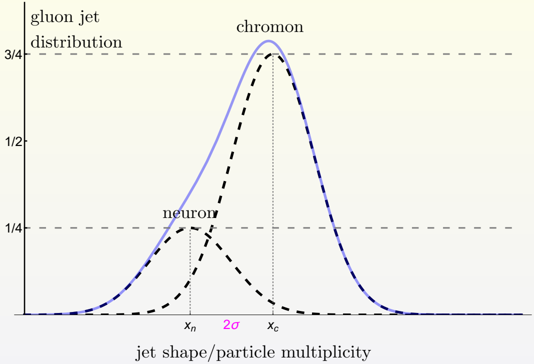

Figure: The expected gluon jet distribution against the jet shape (the sphericity) or the particle multiplicity. Here we have assmed that the distribution is Gaussian, and that the distance between two peaks is 2σ.

- We could also try to separate the neuron jet from the chromon jet in the gluon dominant channels:

$$gluon\ jet: \quad e^+e^- \to H \to gg$$
$$gluon\ enriched\ jet: \quad pp \to dijet$$

- Experimental verification of two different gluon jets becomes more important than the confirmation of the gluon jet.

No New Experiment!

B. Quark and Chromon Model: Chromoballs and Mixed States

- The Abelian decomposition generalizes the quark model to the quark and chromon model which provides a clear picture of glueballs and their mixing with quarkoniums.

- Nevertheless we can make a systematic mixing analysis of chromoball-quarkonium in 0^{++}, 2^{++}, and 0^{-+} sectors below 2 GeV.

- The result shows that $f_0(1500)$ in the 0^{++} sector, $f_2(1950)$ in the 2^{++} sector, $\eta(1405)$ and $\eta(1475)$ in the 0^{-+} sector could be identified as predominantly the glueball states.

C. Monoball: Vacuum Fluctuation of Monopole Condensation

- The monopole condensation could generate the quantum fluctuation. This suggests the existence of at least one monoball, the O^{++} vacuum fluctuation mode. A possible candidate is $f_0(500)$.

- Unlike all other hadrons, it originates from the QCD vacuum. This makes the experimental verification of the monoball an important issue in QCD.

Summary

- The Abelian decomposition is not just a mathematical proposition. It reveals the important hidden structures which simplifies the QCD dynamics greatly.
- It predicts two types of gluons, decomposes the Feynman diagram, simplifies the gauge symmetry, generalizes the quark model, and allows us to prove the monopole condensation.
- This makes the experimental verification of neuron and chromon jets the most urgent issue in QCD.

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