String Theory as a Paradigm

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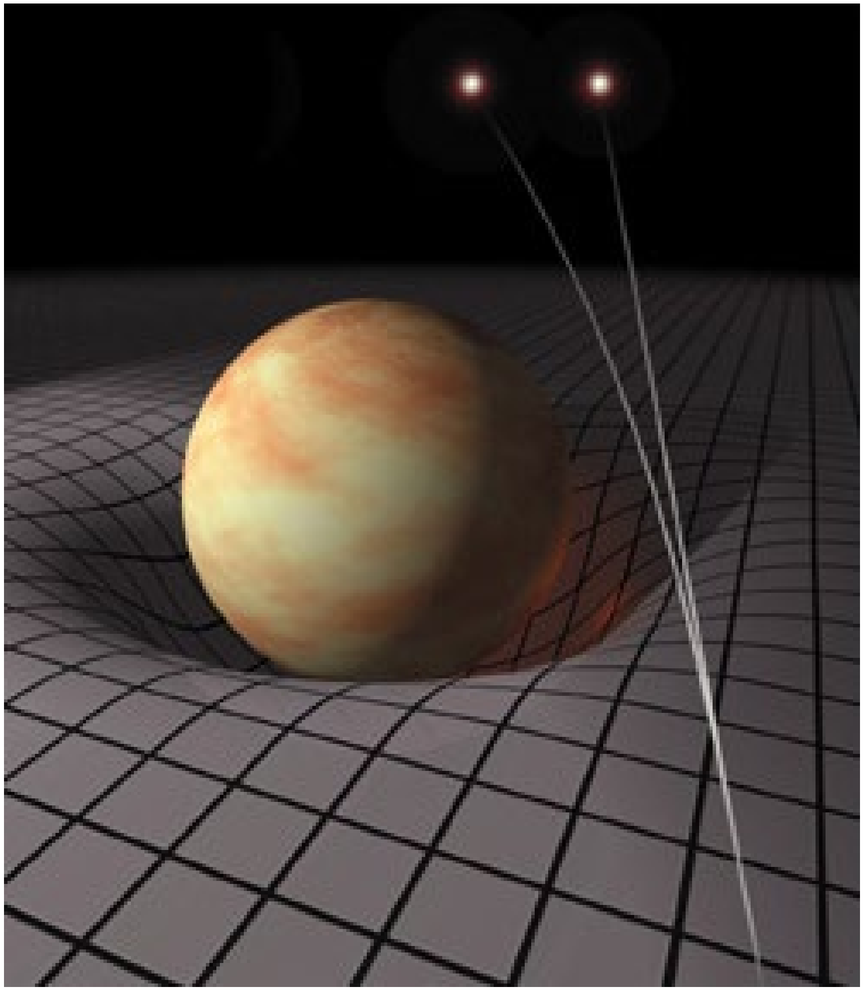
(why not quantum) general relativity?

why strings?

string theory as a paradigm

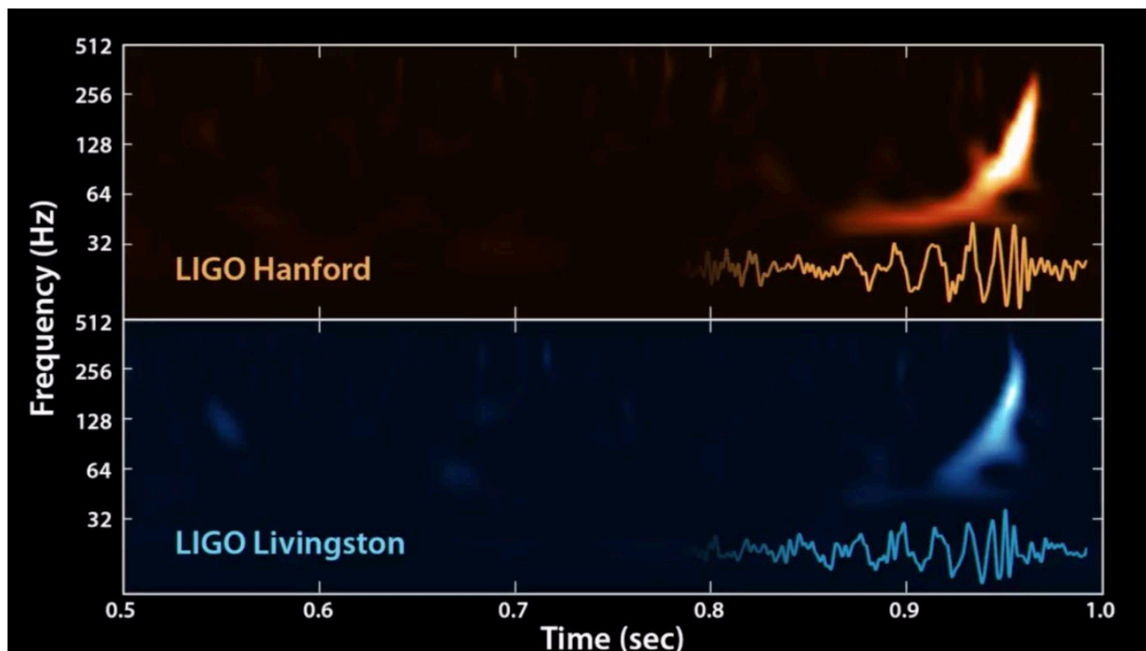
the story starts from Einstein ~ 1915

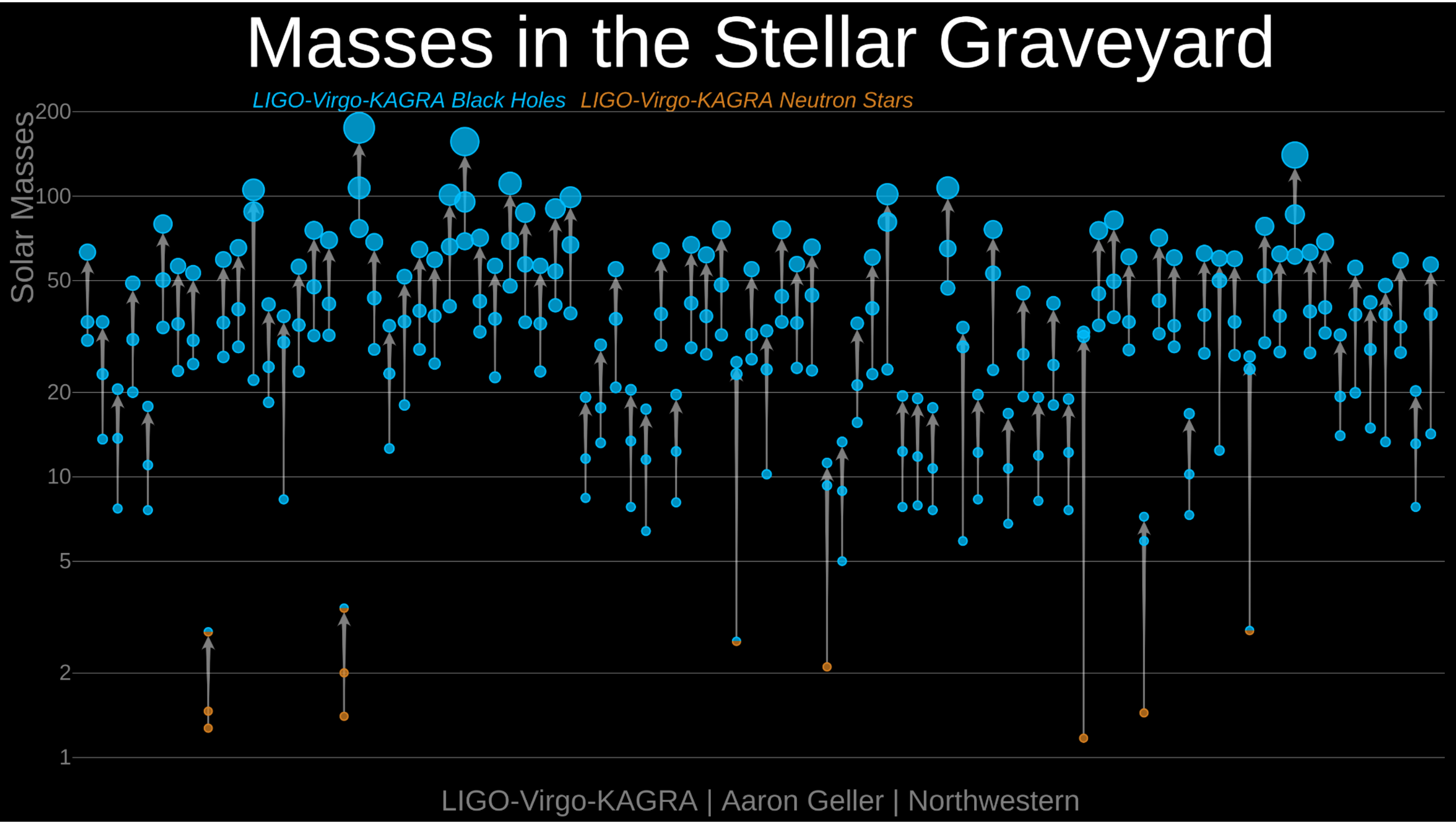


$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

LIGO Livingston

gravitational shock wave from
a binary black hole merger
~1.3 billion light years away
September 14, 2015





November 7, 2021, the LIGO-Virgo-KAGRA Collaboration this theory famously resisted quantization, however why? how?

$$S_{\mathrm{EH}}^{\mathrm{Euclidean}} = -\frac{1}{16\pi G_N}\int \sqrt{g^E}\,R^E$$

why? how?

$$S_{\text{EH}}^{\text{Euclidean}} = -\frac{1}{16\pi G_N}\int \sqrt{g^E}R^E \qquad \propto -L^{d-2}$$

why? how?

$$S_{\text{EH}}^{\text{Euclidean}} = -\frac{1}{16\pi G_N}\int \sqrt{g^E} R^E \qquad \propto \; -L^{d-2}$$

$$\int [Dg]\; e^{-S_{\text{EH}}^{\text{Euclidean}}/\hbar} \qquad \sim \qquad dL\; e^{+\# L^{d-2}/\hbar}$$

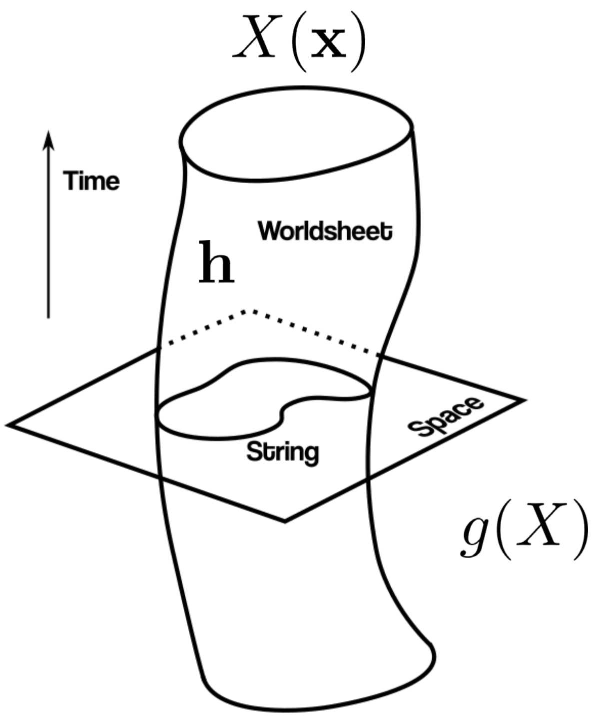
because the path integral is undefinable due to the wrong sign kinetic term for the "size"

$$S_{\rm EH}^{\rm Euclidean} = -\frac{1}{16\pi G_N}\int \sqrt{g^E}R^E$$

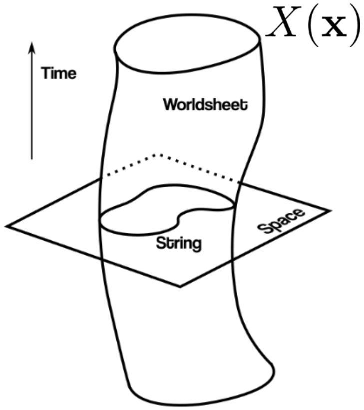
$$-\sqrt{g}\,R(g)\,\Big|_{g=e^{2\Phi}\hat{g}} = -\sqrt{\hat{g}}\,e^{(d-2)\Phi}\left(\hat{R} - 2(d-1)\hat{\nabla}^2\Phi - (d-1)(d-2)(\hat{\nabla}\Phi)^2\right)$$

$$\simeq \sqrt{\hat{g}}\,e^{(d-2)\Phi}\left(-\hat{R} - (d-1)(d-2)(\hat{\nabla}\Phi)^2\right)$$

why string ?



$$S_{\text{string}} = S_{\text{Polyakov}} \equiv -\frac{1}{4\pi\alpha'}\int d\mathbf{x}^2 \,\sqrt{-\det\mathbf{h}}\;\mathbf{h}^{ij}\,g_{\mu\nu}(X)\,\boldsymbol{\partial}_i X^{\mu}\boldsymbol{\partial}_j X^{\nu}$$

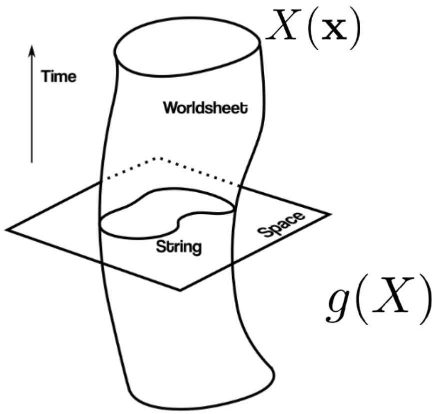


relativistic strings

$$S_{\text{string}} = S_{\text{N.G.}} \equiv -\frac{1}{2\pi\alpha'}\int d\mathbf{x}^2\,\sqrt{-\det\left(\eta_{\mu\nu}\,\boldsymbol{\partial}_i X^\mu \boldsymbol{\partial}_j X^\nu\right)}$$

an obvious generalization for strings

$$S_{\text{particle}} = -mc\int d\boldsymbol{\tau}\,\sqrt{c^2\dot{t}^2 - \dot{x}^2}$$

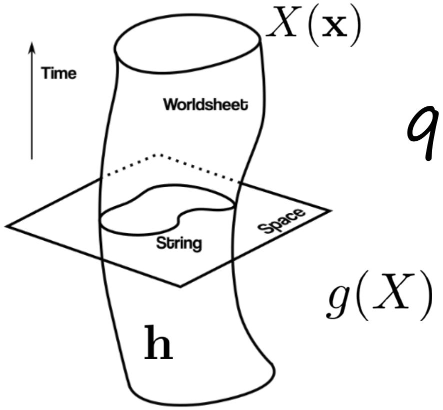


relativistic strings

$$S_{\text{string}} = S_{\text{N.G.}} \equiv -\frac{1}{2\pi\alpha'}\int d\mathbf{x}^2\,\sqrt{-\det\left(g_{\mu\nu}(X)\,\boldsymbol{\partial}_i X^\mu \boldsymbol{\partial}_j X^\nu\right)}$$

an obvious generalization | *for strings in the curved spacetime*

$$S_{\text{particle}} = -mc\int d\boldsymbol{\tau}\,\sqrt{-g_{\mu\nu}(x)\,\dot{x}^\mu \dot{x}^\nu}$$



quantizable relativistic strings

$$S_{\text{string}} = S_{\text{N.G.}} \equiv -\frac{1}{2\pi\alpha'}\int d\mathbf{x}^2\,\sqrt{-\det\left(g_{\mu\nu}(X)\,\boldsymbol{\partial}_i X^\mu \boldsymbol{\partial}_j X^\nu\right)}$$

the same? yes, classically

$\mathbf{h}_{ij}$ equation of motion

$$S_{\text{string}} = S_{\text{Polyakov}} \equiv -\frac{1}{4\pi\alpha'}\int d\mathbf{x}^2\,\sqrt{-\det\mathbf{h}}\;\mathbf{h}^{ij}\,g_{\mu\nu}(X)\,\boldsymbol{\partial}_i X^\mu \boldsymbol{\partial}_j X^\nu$$

no different from how we arrive at geodesics

for star trajectories in general relativity

$$S_{\text{particle}} = -mc \int d\boldsymbol{\tau} \, \sqrt{-g_{\mu\nu}(x)\, \dot{x}^{\mu} \dot{x}^{\nu}}$$

the same? yes, classically ↕ $\mathbf{h}$ equation of motion

$$S'_{\text{partice}} = \frac{mc}{2} \int d\tau \, \mathbf{h} \, g_{\mu\nu}(X) \, \dot{X}^{\mu} \dot{X}^{\nu} - \mathbf{h}^{-1}$$

↓

geodesics

the worldsheet metric $\mathbf{h}_{ij}$ is not a physical thing

$$S_{\text{string}} = S_{\text{N.G.}} \equiv -\frac{1}{2\pi\alpha'}\int d\mathbf{x}^2\,\sqrt{-\det\left(g_{\mu\nu}(X)\,\boldsymbol{\partial}_i X^\mu \boldsymbol{\partial}_j X^\nu\right)}$$

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this equivalence must hold at quantum level as well

$$0 = \frac{\delta}{\delta \mathbf{h}} \int [DX] \, e^{-S_{\text{Polyakov}}(\mathbf{h}, g(X); X)/\hbar}$$

which translates to vanishing "Weyl anomaly"

if the general coordinate invariance is respected

$$0 = \frac{\delta}{\delta \mathbf{h}} \int [DX] \, e^{-S_{\rm Polyakov}(\mathbf{h}, g(X); X)/\hbar}$$

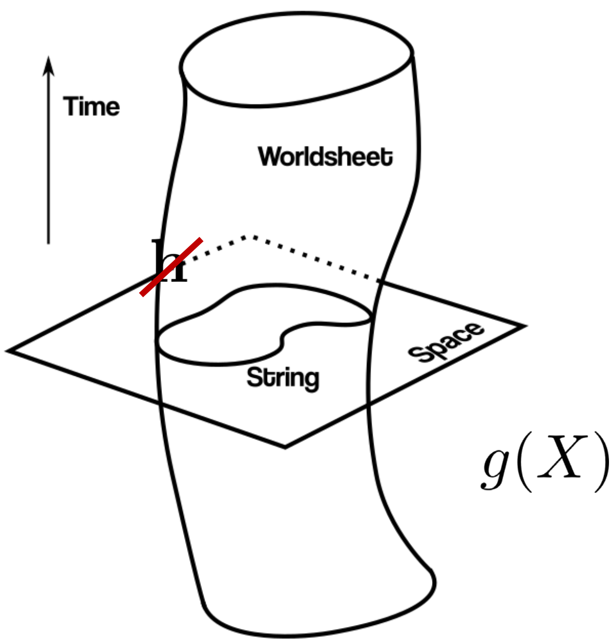
$\Updownarrow$ $\mathbf{h}_{ij} = e^{2\rho} \boldsymbol{\delta}_{ij}$

$$0 = \frac{\delta}{\delta \rho} \int [DX] \, e^{-S_{\rm Polyakov}(\mathbf{h}, g(X); X)/\hbar}$$

this self-consistency, surprisingly, produces a spacetime Einstein equation

$$0 = \frac{\delta}{\delta \mathbf{h}} \int [DX]\, e^{-S_{\mathrm{Polyakov}}(\mathbf{h}, g(X); X)/\hbar}$$

$$0 = R(g)_{\mu\nu} + \cdots$$

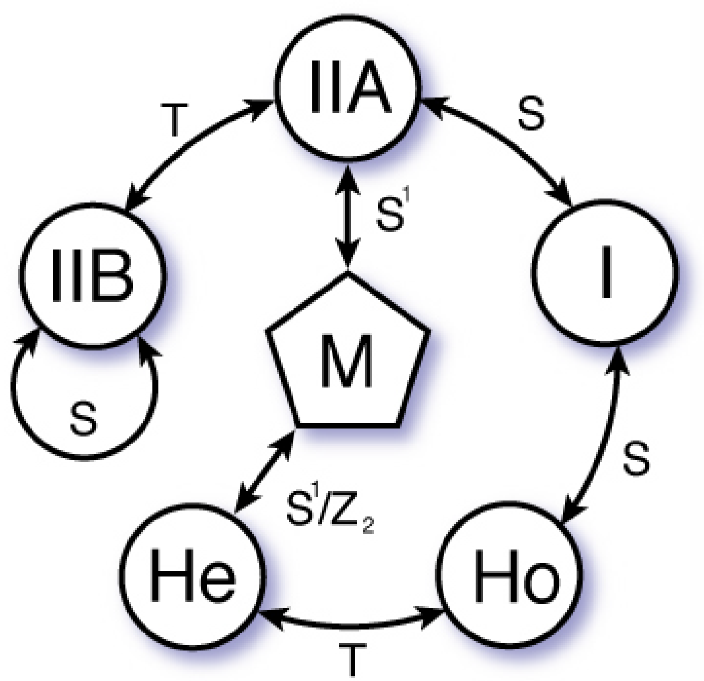


fundamental strings in generic curved spacetime
are quantum mechanically nonsense

strings can be quantized only if the spacetime
obeys some version of Einstein gravity
implied by quantized strings themselves

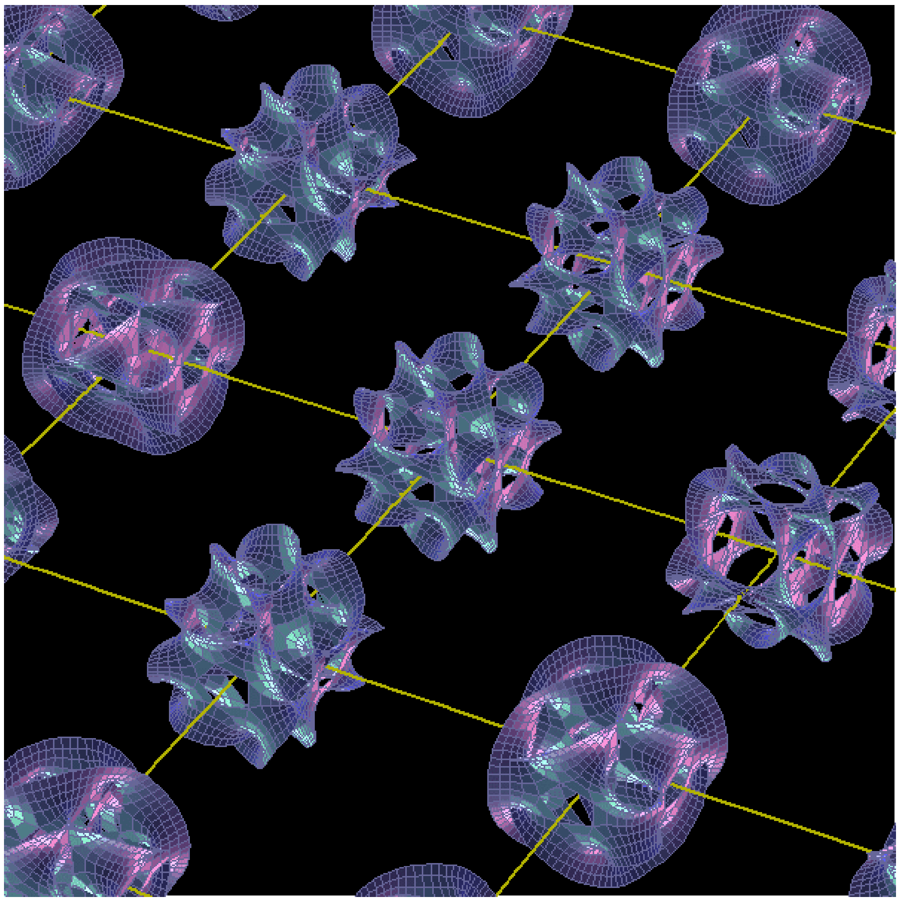
among other necessary conditions
on equal footing as the above is $d=10$

superstring theories (and M-theory)



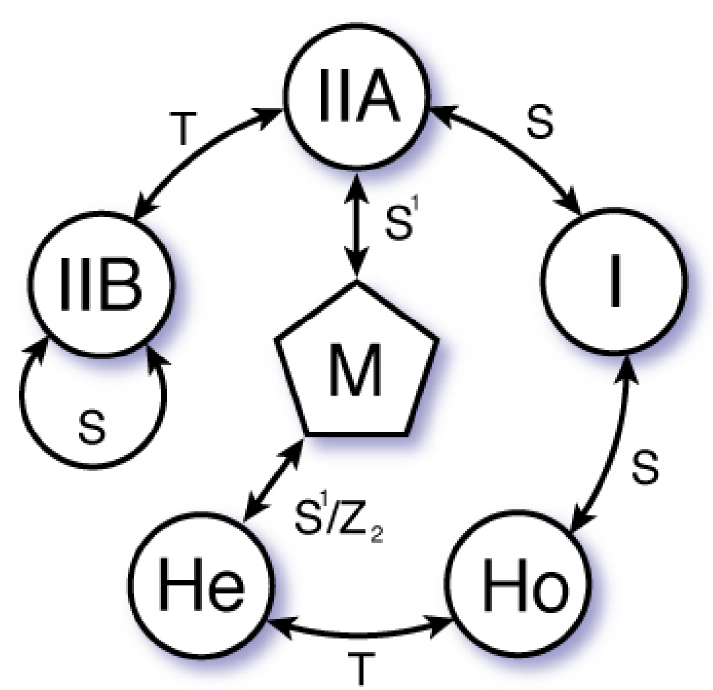
how are 4d worlds possibly realized ?

$$10^{-13}cm \gg l_{size} > 10^{-33}cm$$



$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi\kappa_{9+1}^2 T_{\mu\nu}$$

d=4 manifestations of these five superstring theories are infinitely diverse → creates a new paradigm on par with quantum field theories



where are we?

do we understand quantum gravity & quantum black holes?

are the world physics/cosmology realizable?

e.g., landscape & swampland

universal UV completions of quantum field theories

a toolbox for enumerative geometry

and ultimately ...

do we now understand quantum gravity, or string theory in its most fundamental form?

do we understand quantum black holes?

black hole micro-states, sort of / information puzzle, not quite yet

is the gravity (or closed string) induced (from open string)?

ultimately, is there a 2nd quantized theory of some kind that encompasses all superstring theories and the M-theory?

real world physics? a real world cosmology?

Standard Model and its "UV completions (= stringy BSM)"

landscape & swampland?

stringy signatures in visible universe?

stringy inflation, tensor-to-scalar ratio, reheating,

exotic dark matter content, detectible topological defects, etc

how numerous are de Sitter solutions?

is the dark energy puzzle a scientific problem?

universal UV completions of supersymmetric quantum field theories

geometrical engineering for supersymmetric gauge theories

dualities & nonperturbative structures

localization & topological partition function

(exotic) interacting conformal field theories

holography for strongly coupled dynamics

a toolbox for enumerative geometry

mirror symmetry and Gromov-Witten invariants

wall-crossing and Donaldson-Thomas invariants

localization and topological partition functions

beyond all these ?

and more ultimately ... what replaces this?

$$\int [Dg][D\cdots]\, e^{iS(g,\cdots)/\hbar}$$