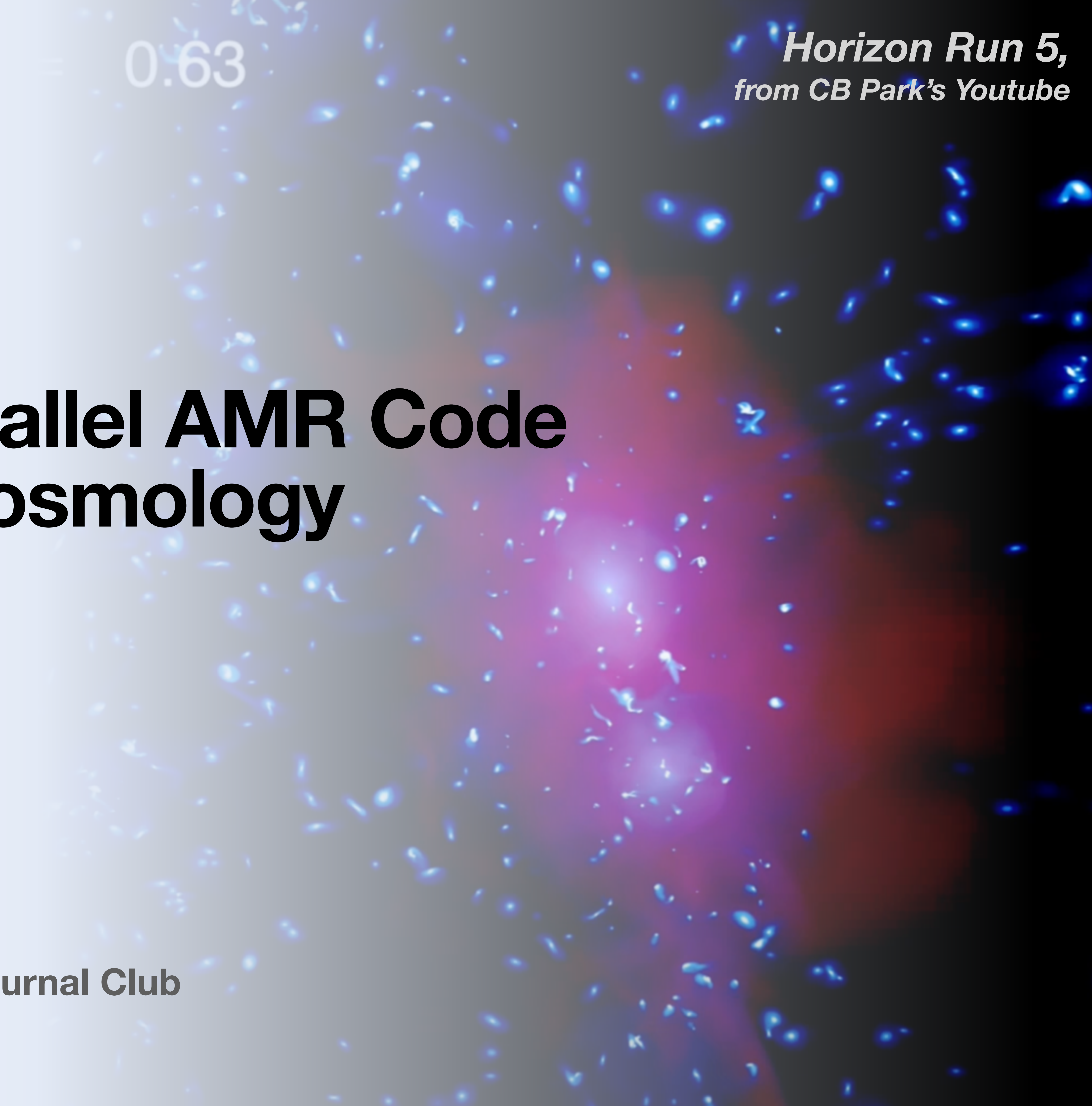
Nyx: A Massively Parallel AMR Code for Computational Cosmology

Ann S. Almgren et al., 2013, ApJ, 765, 39



Horizon Run 5,
from CB Park's Youtube

Presented by Hannah Jhee @ Cosmology Journal Club

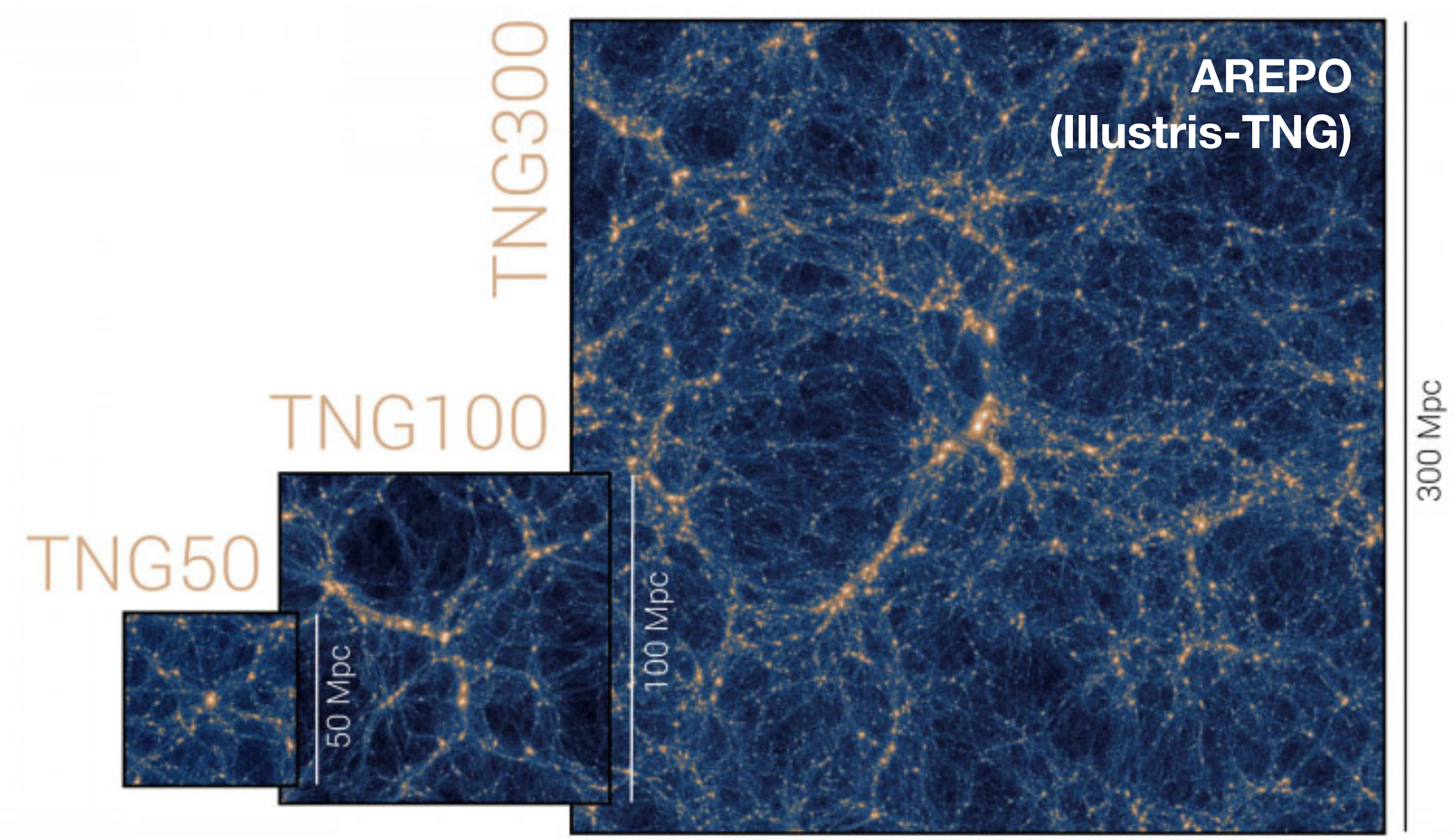
Cosmological Simulations

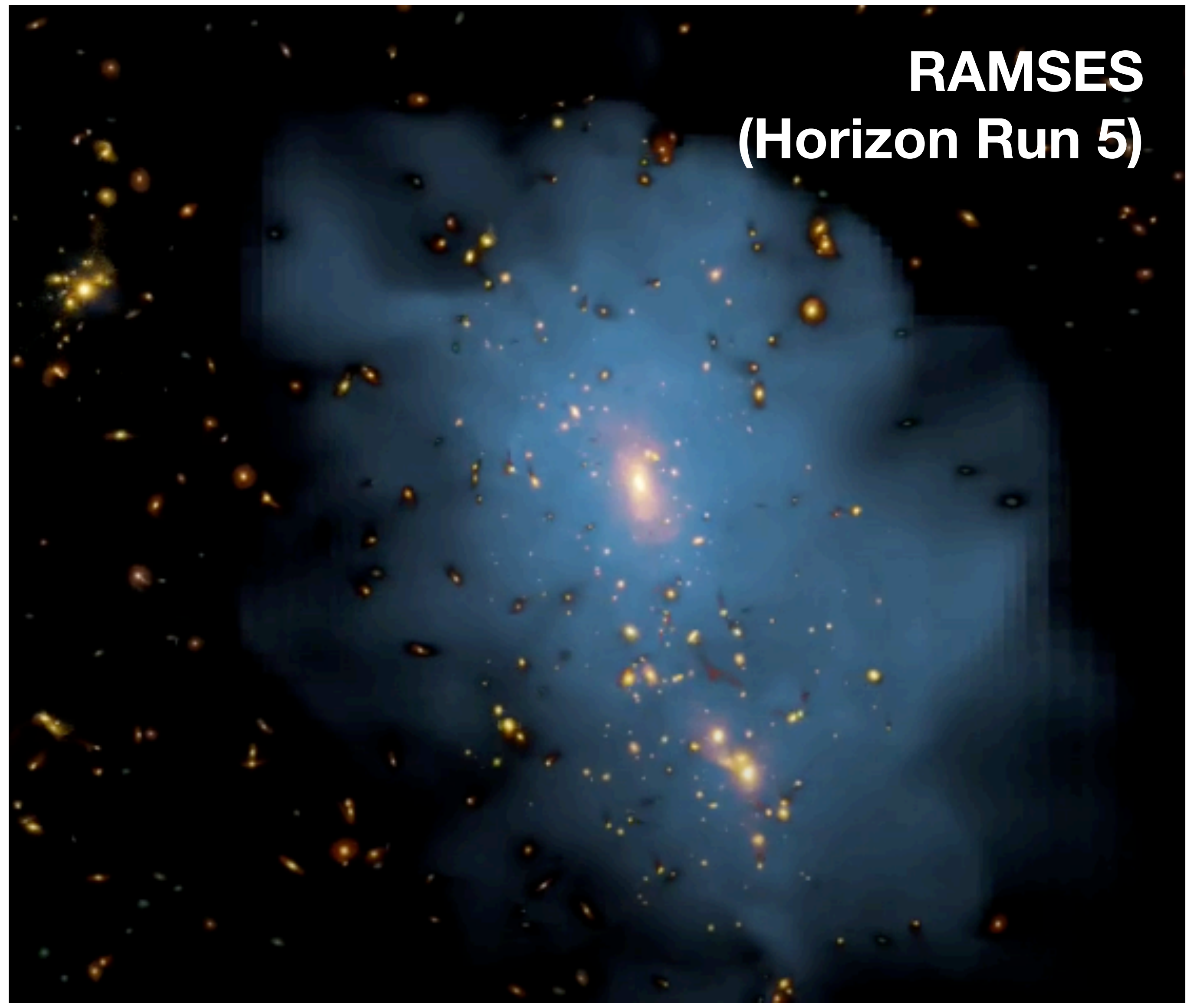
Cosmological Simulations

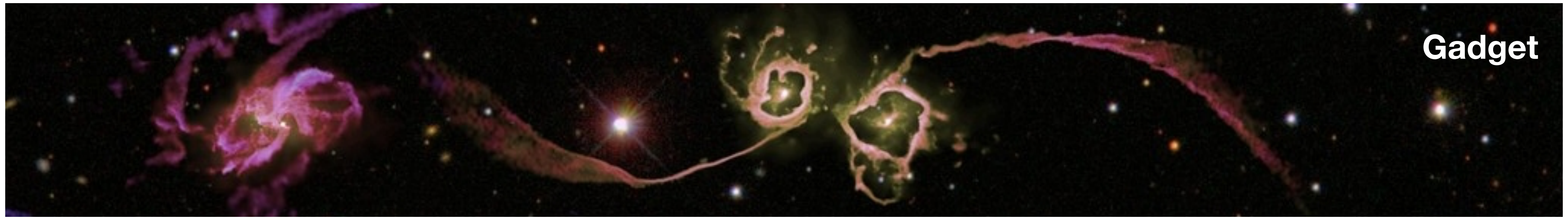
matter contents

DM-only

Hydrodynamics







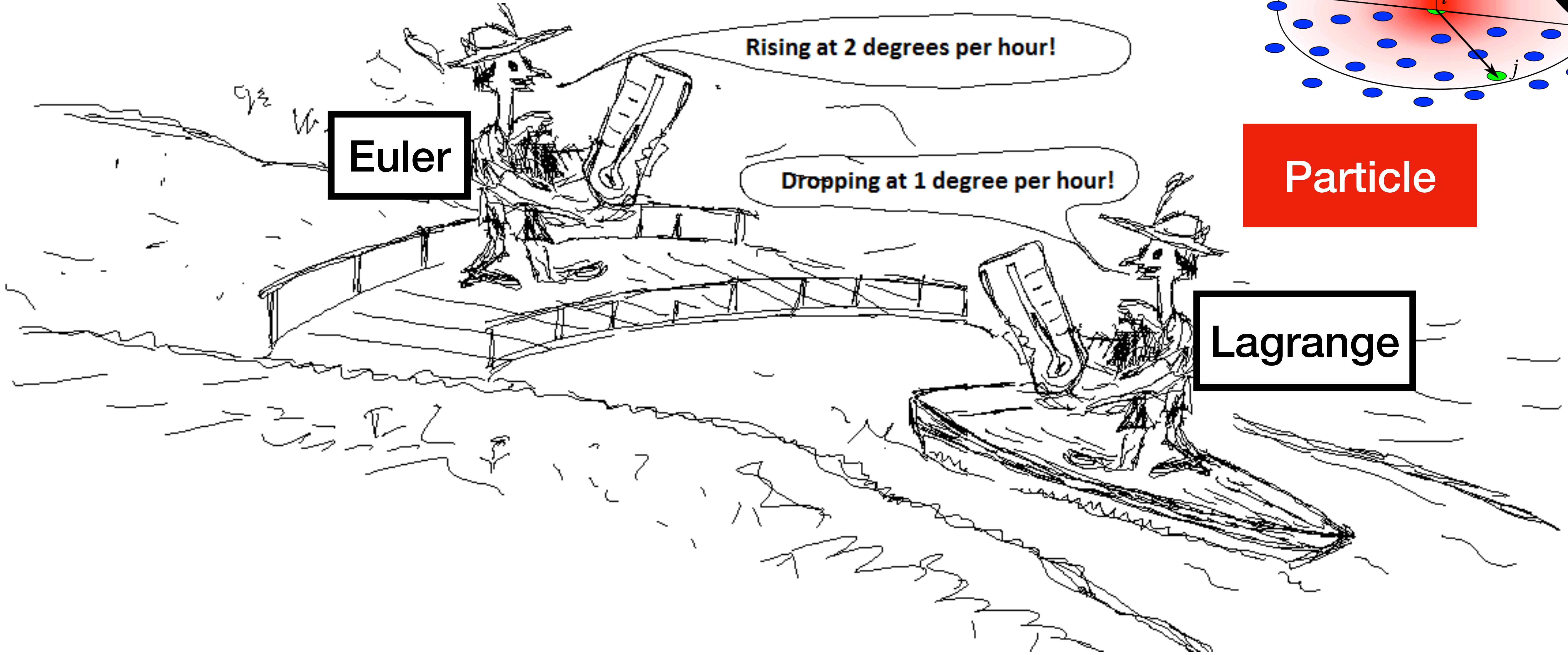
Cosmological Simulations

points of view

Eulerian

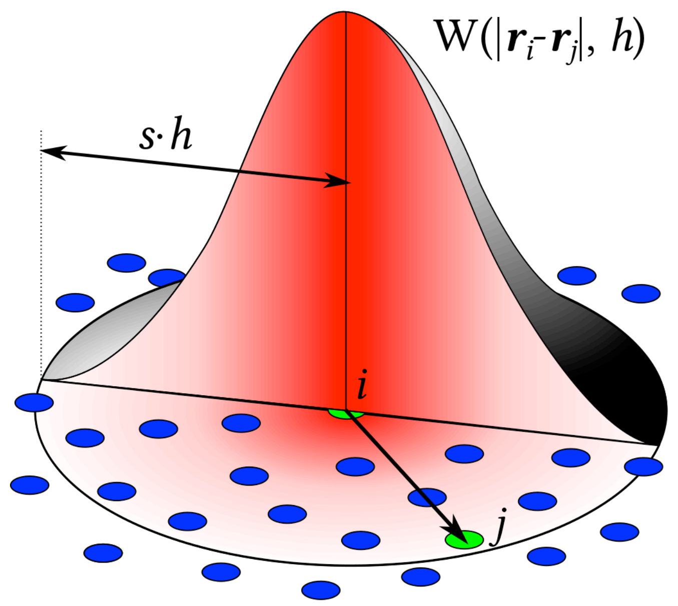
Lagrangian

Mesh



Particle

Smoothed Particle Hydrodynamics

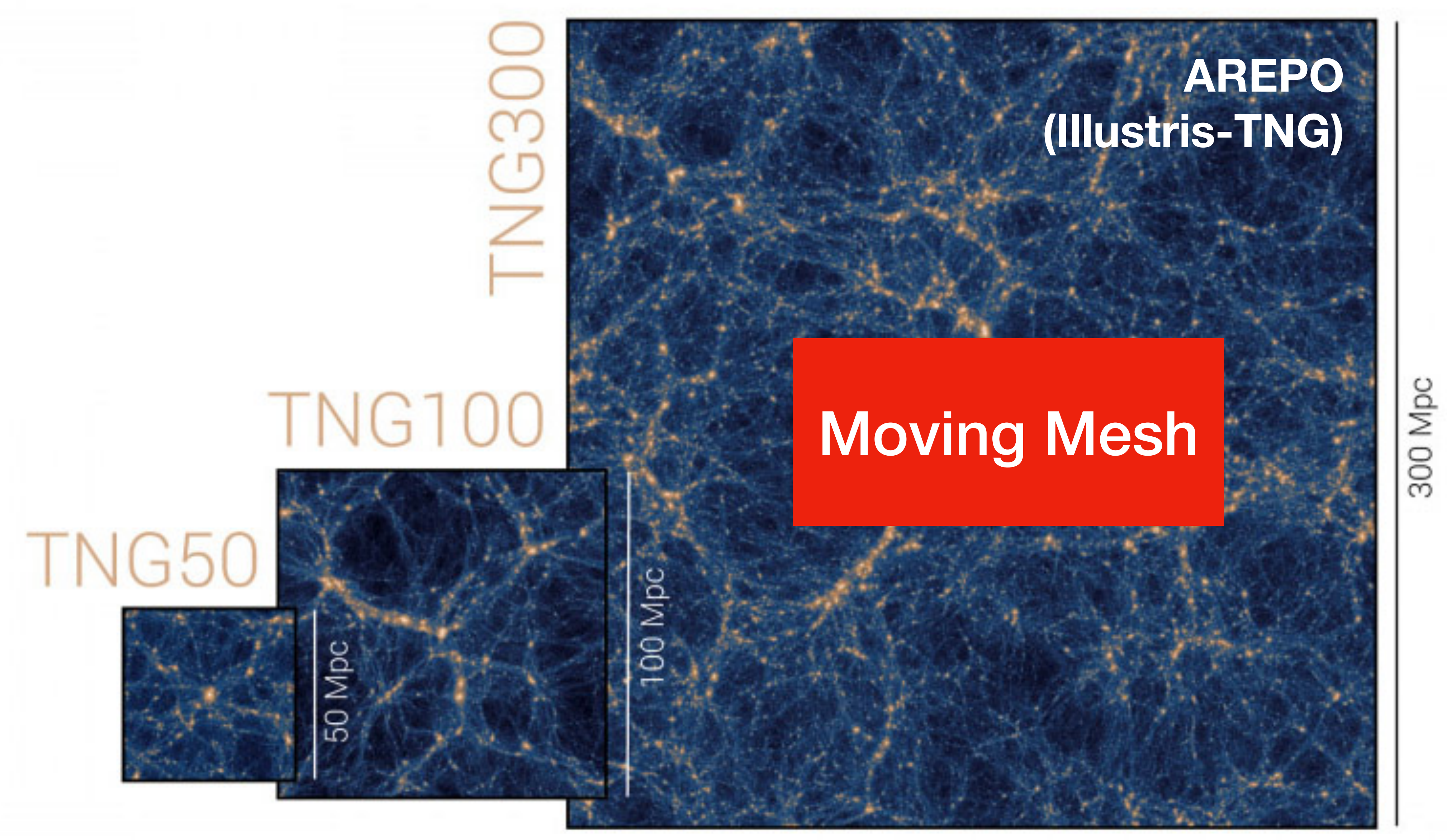


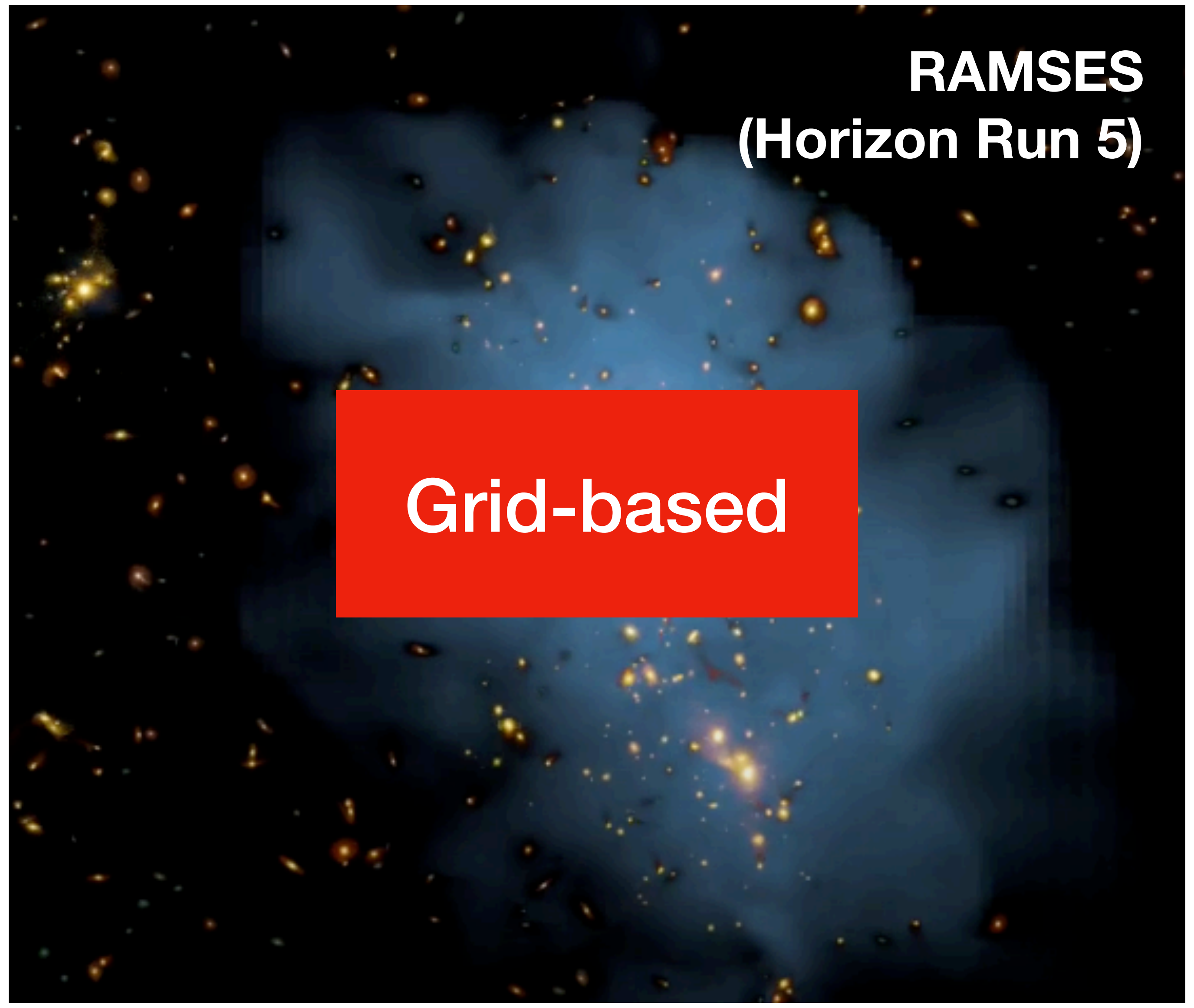
Cosmological Simulations

points of view

Eulerian

Lagrangian





Basic Equations

Basic Equations

into codes

Expanding Universe

Gas dynamics

Dark Matter(CDM)

Self Gravity

Before evolving gas/dm elements, first expand the universe according to Friedmann eq.

$$\frac{d}{dt}\ln a = \frac{\dot{a}}{a} = H_0\sqrt{\frac{\Omega_0}{a^3} + \Omega_\Lambda}$$

Solve this equation with m 2nd-order Runge-Kutta Method, with m satisfying

$$\frac{|a_m^{n+1} - a_{2m}^{n+1}|}{a_m^{n+1}} < 10^{-8}$$

Δt is determined following the rules as $\Delta t = \min(\Delta t_{\rm hyp}, \Delta t_{\rm dm}, \Delta t_a)$:

$$\Delta t_{\rm hyp} = \sigma^{\rm CFL,hyp} \min_{i=1,\dots3}\left[\frac{a\Delta x}{\max_{\vec{x}}|\vec{U}\cdot\hat{e}_i| + c}\right] \qquad \Delta t_{\rm dm} = \sigma^{\rm CFL,dm} \min_{i=1,\dots3}\left[\frac{a\Delta x}{\max_j|\vec{u}_j\cdot\hat{e}_i|}\right]$$

Basic Equations

into codes

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Equations for compressible hydrodynamics(Euler eqs.)

$$\frac{\partial \rho}{\partial t} + \frac{1}{a}\vec{\nabla} \cdot (\rho \vec{U}) = 0$$

Mass Conservation

$$\frac{\partial (a\rho_b \vec{U})}{\partial t} + (\rho_b \vec{U} \cdot \vec{\nabla})\vec{U} = -\vec{\nabla} P + \rho_b \vec{g}$$

Momentum Conservation

$$\frac{\partial (a^2\rho_b e)}{\partial t} = -a\vec{\nabla} \cdot (\rho_b \vec{U} e) - aP\vec{\nabla} \cdot \vec{U} + a\dot{a}((2-3(\gamma-1))\rho_b e) + a\Lambda_{HC}$$

$$\frac{\partial (a^2\rho_b E)}{\partial t} = -a\vec{\nabla} \cdot (\rho_b \vec{U} E + P\vec{U}) + a\rho_b \vec{U} \cdot \vec{g} + a\dot{a}((2-3(\gamma-1))\rho_b e) + a\Lambda_{HC}$$

Energy Conservation

$$P = \frac{\rho k_B T}{\mu m_H} = (\gamma - 1)\rho u$$

Equation of State (Ideal gas)

Basic Equations

into codes

Expanding Universe

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Self Gravity

Phase-space function $f(\vec{x}, \vec{p})$ is described by Vlasov(Collisionless Boltzmann) equation:

$$\frac{\partial f}{\partial t} + \frac{1}{ma^2}\vec{p} \cdot \vec{\nabla} f - m\vec{\nabla}\phi \cdot \frac{\partial f}{\partial \vec{p}} = 0$$

Rather than solving the equation, we place N particles using Monte-carlo sampling evolving acoording to:

$$\frac{d\vec{x}_i}{dt} = \frac{1}{a}\vec{u_i} \qquad \frac{d(a\vec{u}_i)}{dt} = \vec{g}(\vec{x}, t)$$

This assertion is validated from Liouville's theorem.

Basic Equations
into codes

Expanding Universe

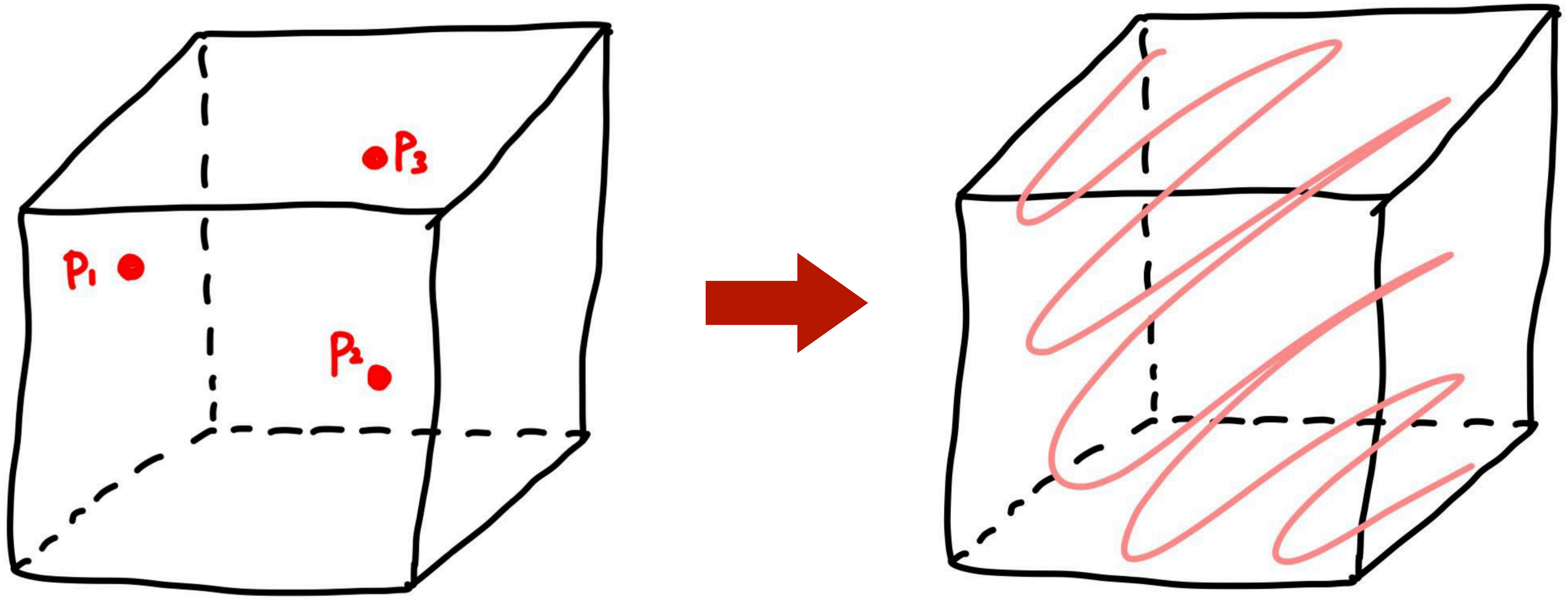
Gas dynamics

Dark Matter(CDM)

Self Gravity

At a given position and time, dm and baryon both contribute to the gravitational fields:

$$\nabla^2 \phi(\vec{x}, t) = \frac{4\pi G}{a}(\rho_b + \rho_{dm} - \rho_0)$$

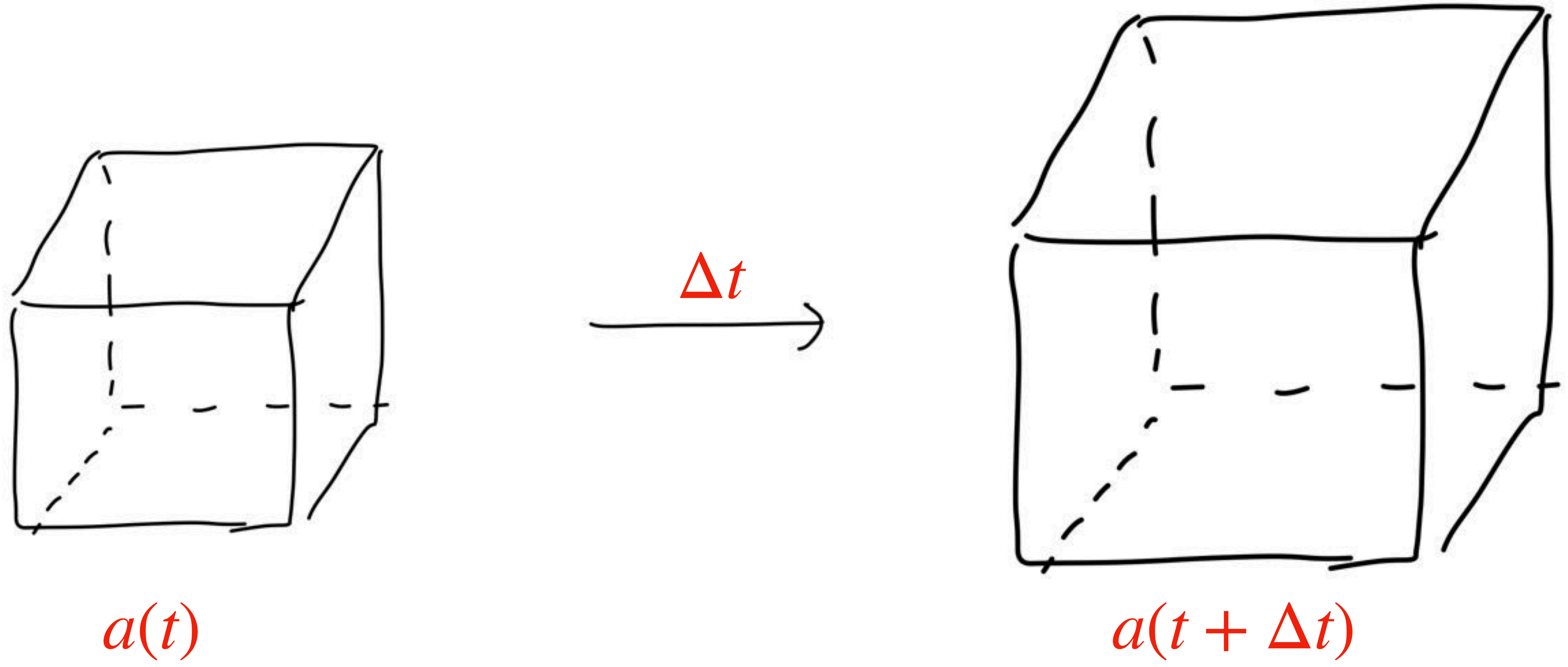


Algorithms

Single-Level Algorithm

straight to the algorithm

1. Calculate the time step Δt, and advance the scale factor $a(t)$.



Single-Level Algorithm

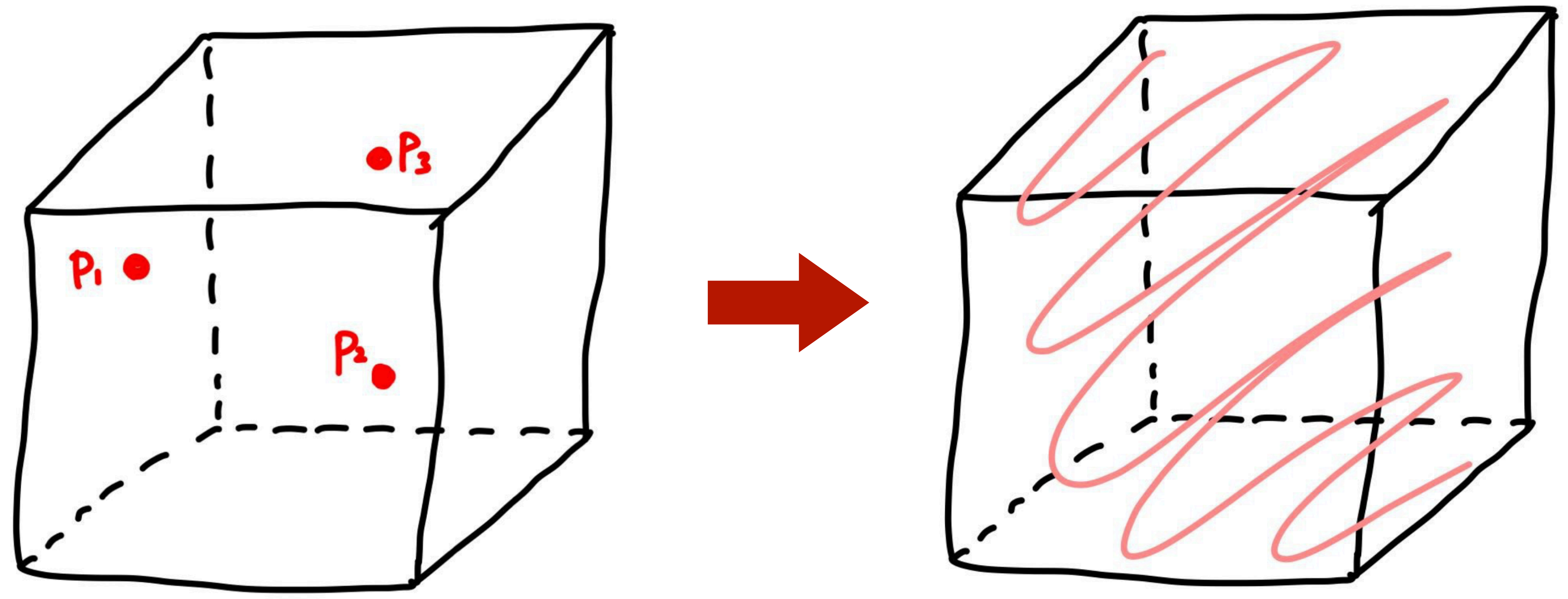
straight to the algorithm

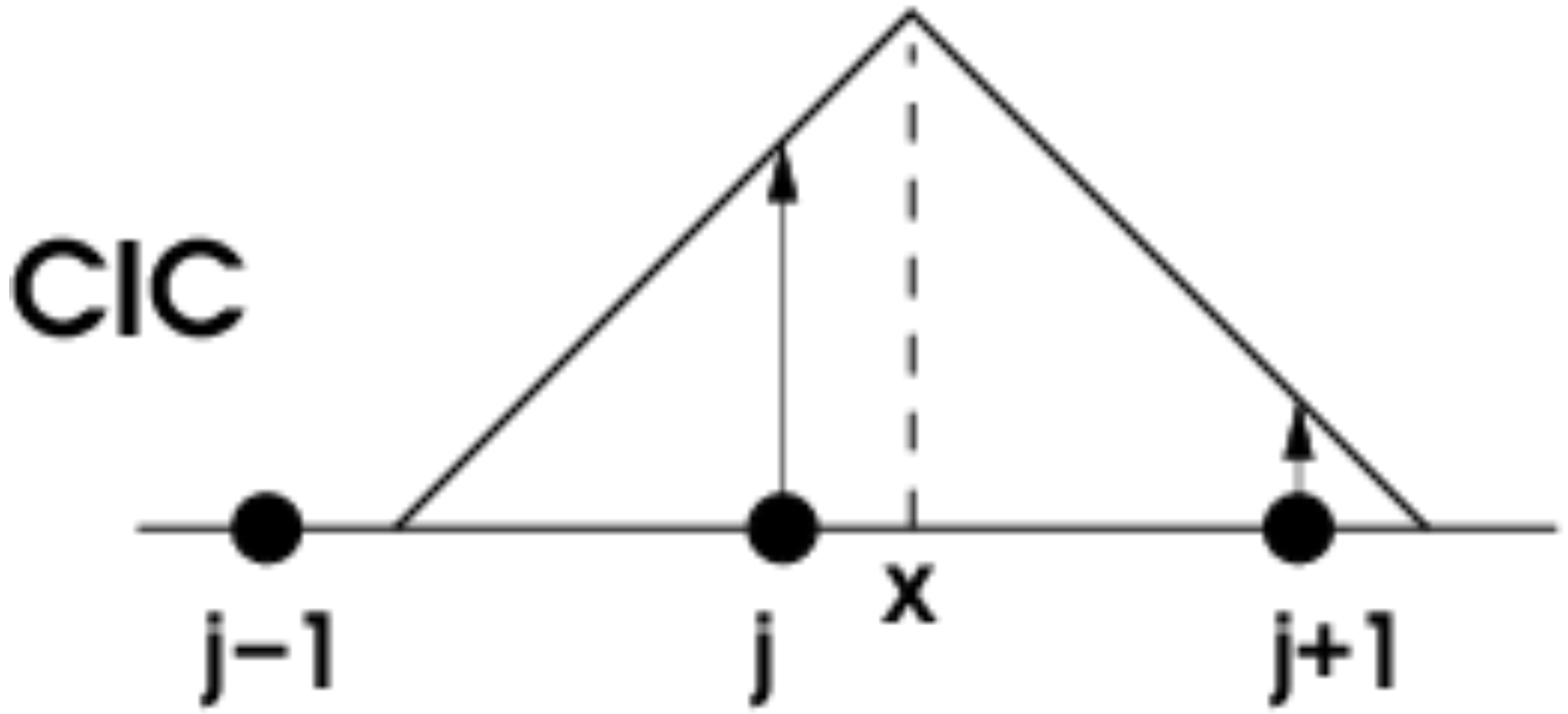
2. Compute ϕ^n and $\vec{g}^n$ using ρ_b^n and ρ_{dm}^n.

Particle-Mesh Method

↓

Cloud-In-Cell(CIC) Scheme





Single-Level Algorithm

straight to the algorithm

3. Advance the state of the gas.

4. Advance the dm particle positions, gravitational fields and velocities according to:

$$\vec{u}_i^{n+1/2} = \frac{1}{a^{n+1/2}} \left((a^n \vec{u}_i^n) + \frac{\Delta t}{2} \vec{g}_i^n \right) \qquad \vec{x}_i^{n+1} = \vec{x}_i^n + \frac{\Delta t}{a^{n+1/2}} \vec{u}_i^{n+1/2} \qquad (a \vec{u}_i)^{n+1} = (a \vec{u}_i)^{n+1/2} + \frac{\Delta t}{2} \vec{g}_i^{n+1}$$

5. Advance the state of the gas.

Predictor-Corrector

값의 대략적인 근사치를 예측한 후(Predictor)
다른 수단을 이용해 초기 근사치를 수정(Corrector)

Strang Splitting

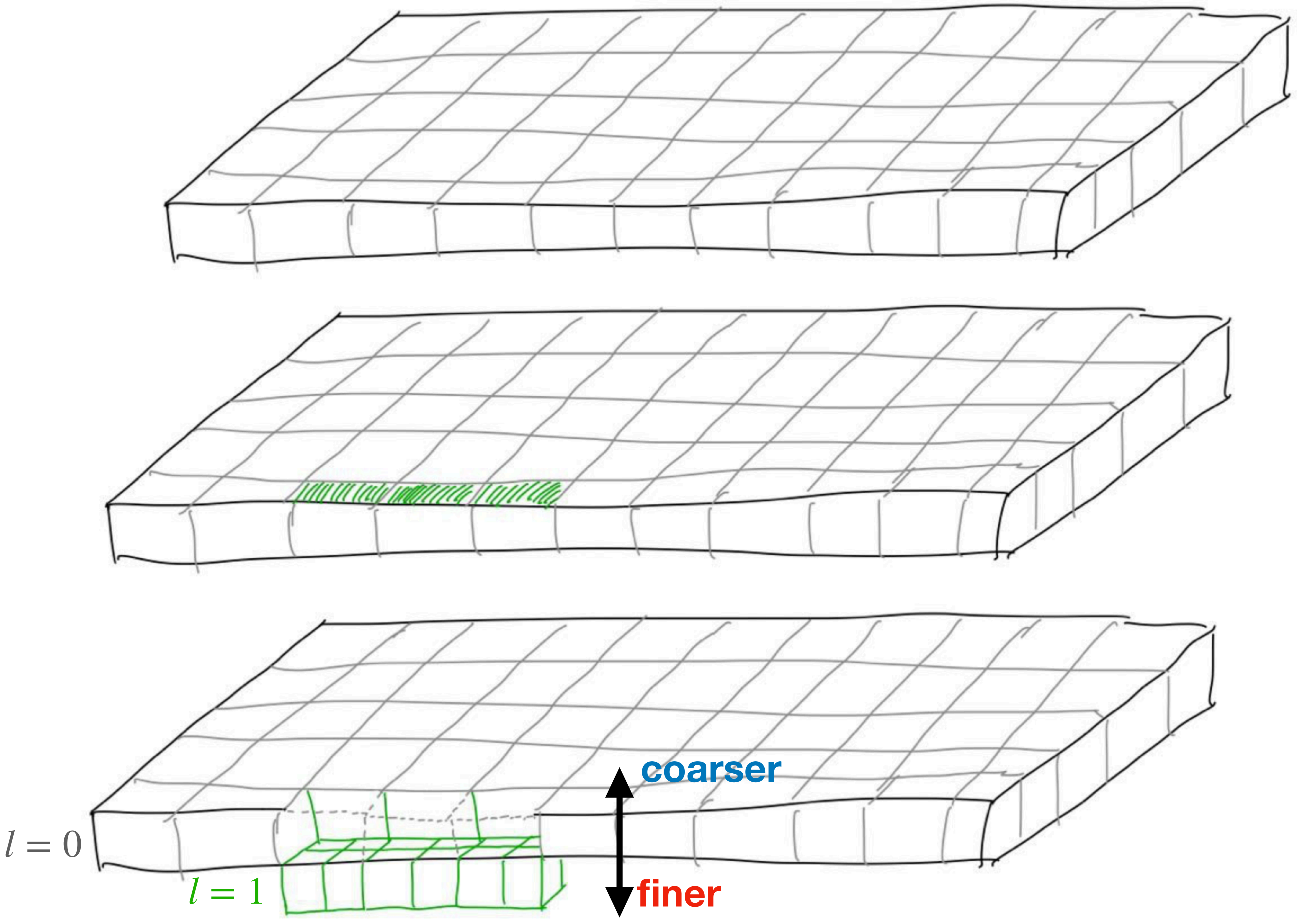
값의 대략적인 근사치를 예측한 후(Predictor)
다른 수단을 이용해 초기 근사치를 수정(Corrector)

Multi-Level Algorithm

Block-structured AMR, Mesh hierarchy

Concepts in Block-structured AMR

- **Grid**
- **Cell (valid, ghost)**
- **Level**



Multi-Level Algorithm

Block-structured AMR이란 무엇인가?

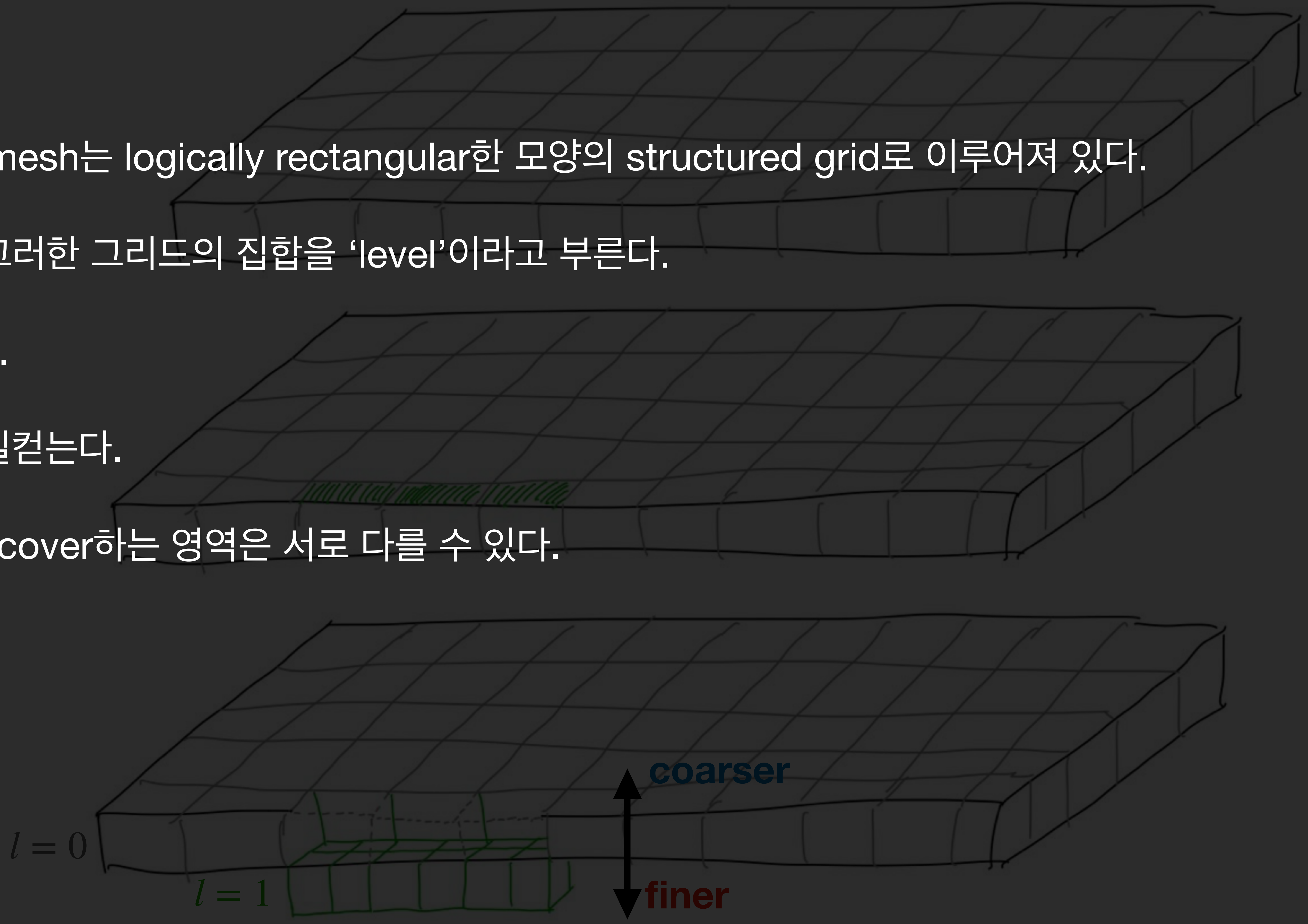
Concepts in Block-structured AMR

Grid

Cell

Level

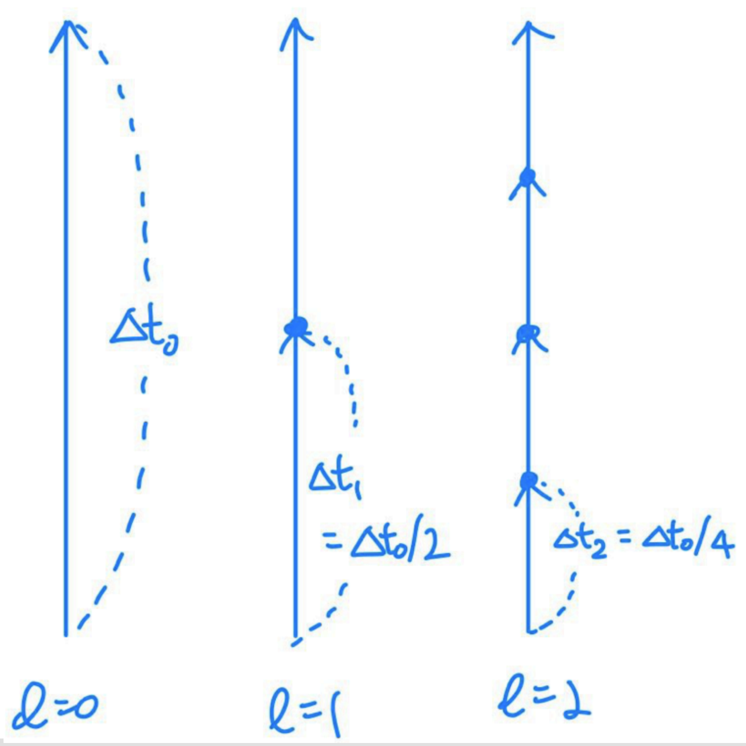
- 시뮬레이션이 이루어지는 전체 영역(domain)을 이루는 mesh는 logically rectangular한 모양의 structured grid로 이루어져 있다.
- 같은 크기의 셀로 이루어진 그리드는 서로 겹치지 않고, 그러한 그리드의 집합을 ‘level’이라고 부른다.
- 가장 coarse한 level은 domain을 모두 cover해야 한다.
- Complete한 mesh hierarchy는 모든 level의 집합을 일컫는다.
- Mesh 데이터와 입자 데이터의 각 레벨에 있는 그리드가 cover하는 영역은 서로 다를 수 있다.



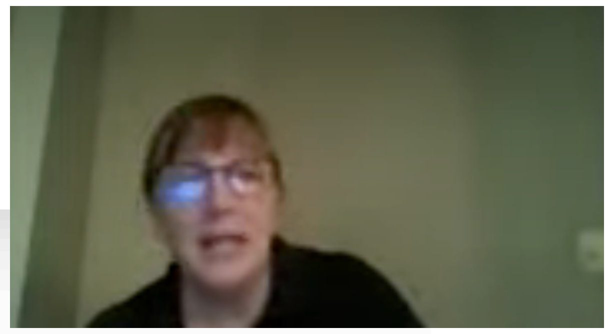
See *https://youtu.be/wE3tdL_p6Ms*

Multi-Level Algorithm

Block-structured AMR, Mesh hierarchy



Ann Almgren



Concepts in Block-structured AMR

Grid

Cell (valid, ghost)

Level

Subcycling

What about Time-Stepping

AMR doesn't dictate the spatial or temporal discretization on a single patch, but we need to make sure the data at all levels gets to the same time.

The main question is:

To subcycle or not to subcycle?

Subcycling in time means taking multiple time steps on finer levels relative to coarser levels.

Non-subcycling:

각 level의 time step이 Δt_a로 정해지는 경우, subcycling을 사용하지 않는 것이 효율적이다.

Subycling:

예) Level 2에 있는 dm 입자가 Level 3에 있는 입자보다 2배 빠르게 움직이고, time step이 Δt_{dm}으로 정해지는 경우, level 2와 3에 대해서는 같은 시간간격 Δt로 진화시키고 level 1은 level 0에 대해 subcycle, level 2는 level 1에 대해 subcycle하는 것이 효율적이다.

Multi-Level Algorithm

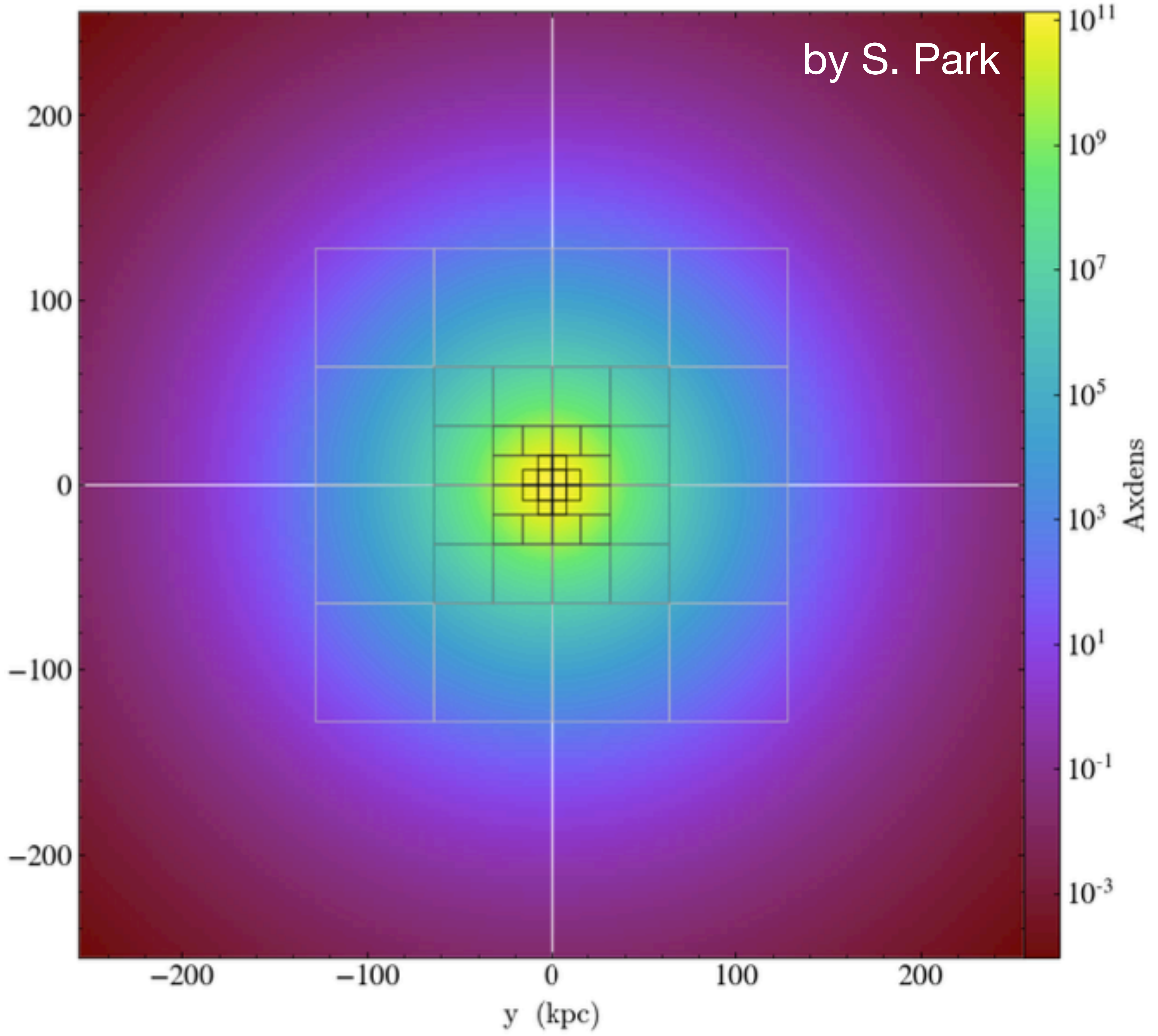
파악하지 못한 것들…

Interpolation의 정확한 방법

어떻게 해서 직육면체 모양의 셀이 생기는지

subcycling을 왜 사용하는지
(정확히 어떤 측면에서 효율적이라는 건지)

Multigrid에서 Poisson solver에 관한 내용



Validation

$\Omega_m = 0.314$
$\Omega_\Lambda = 0.686$
$h = 0.71$
$\sigma_8 = 0.84$
$L = 256\,\mathrm{Mpc/h}$
$m_{\mathrm{dm}} = 1.227 \times 10^{11} M_\odot$

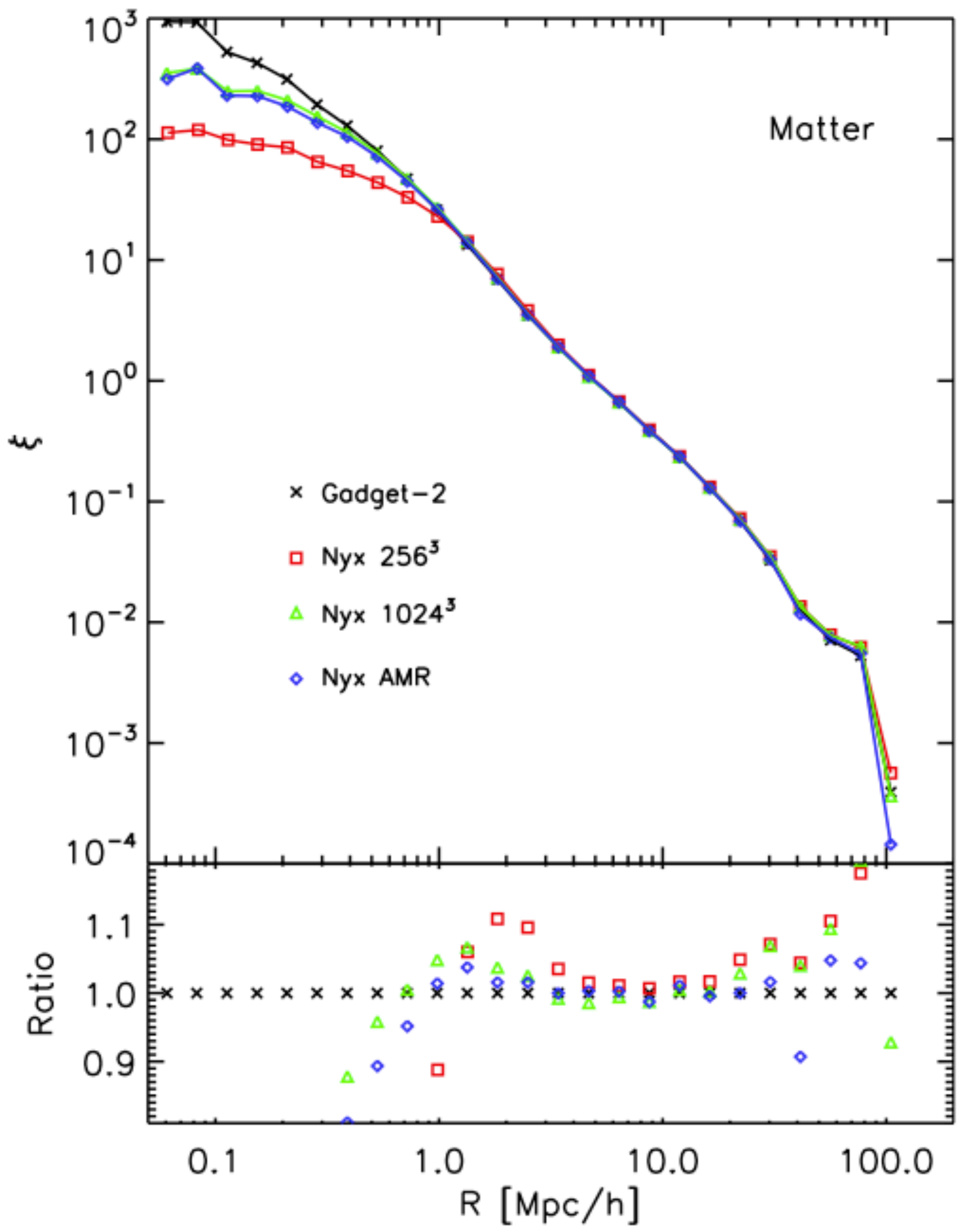
→

Gadget-2
Nyx 256^3 uniform grid
Nyx 1024^3 uniform grid
Nyx 256^3 with 2-level AMR

Validation

Particle :: 2pcf

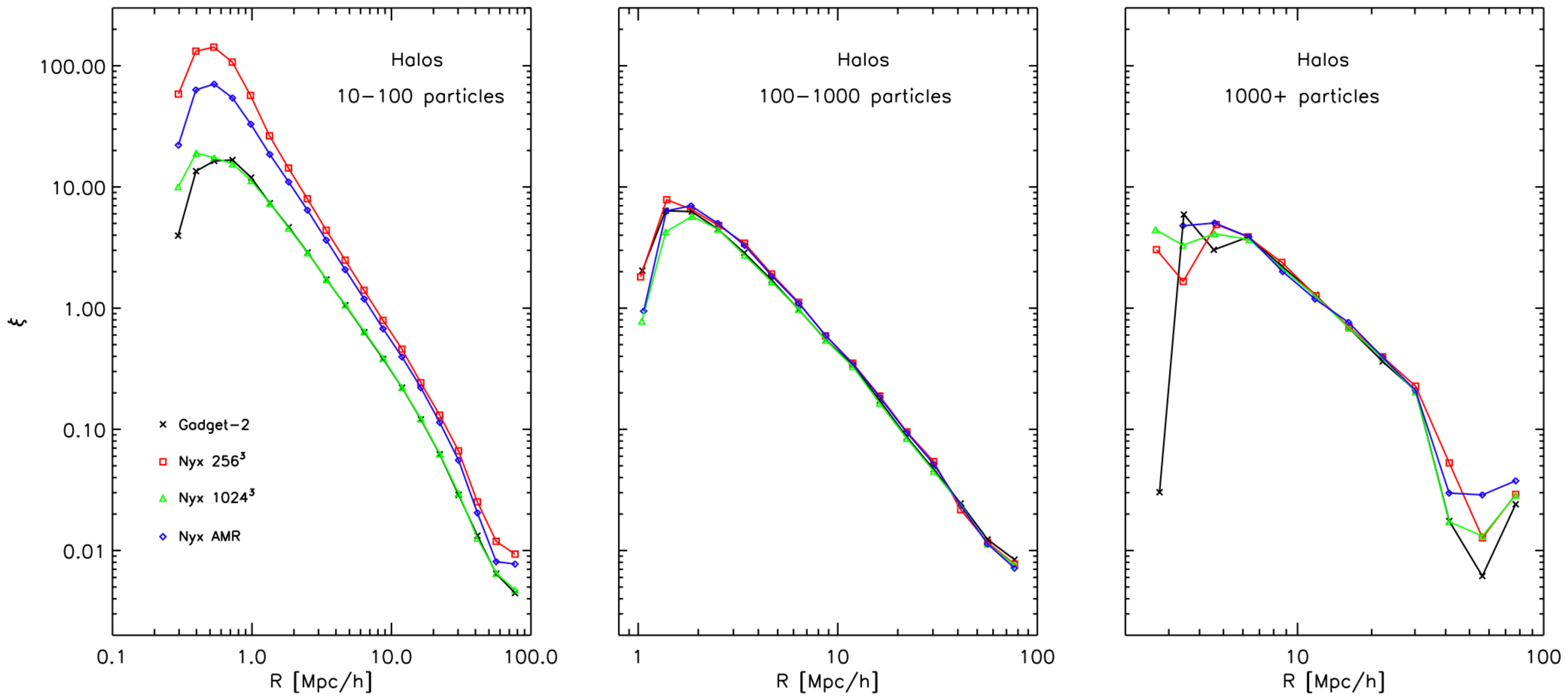
- Stronger match to Gadget-2 as the resolution increases
- AMR(effective 1024) results almost matches uniform 1024

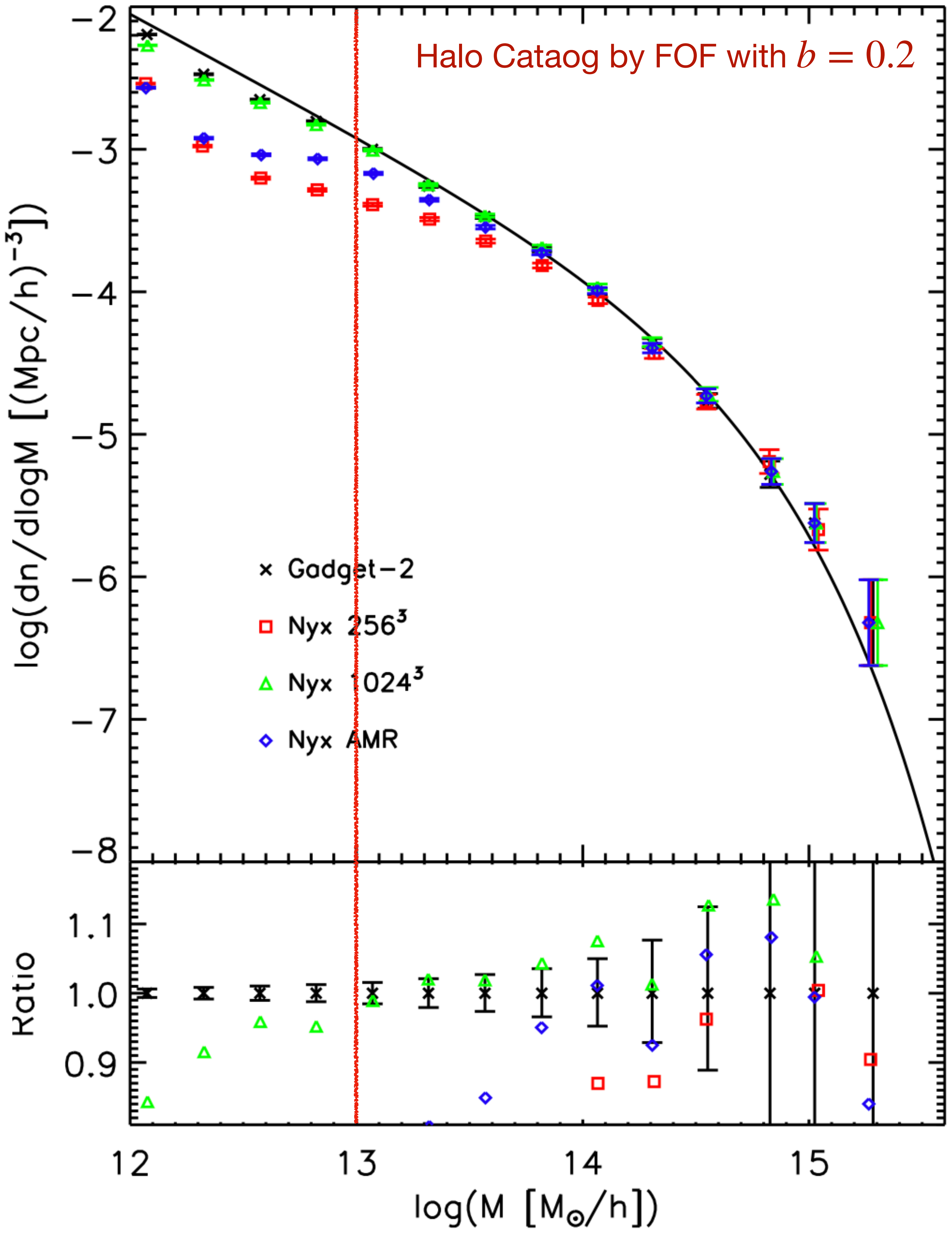


Validation

Halo :: Mass function and 2pcf

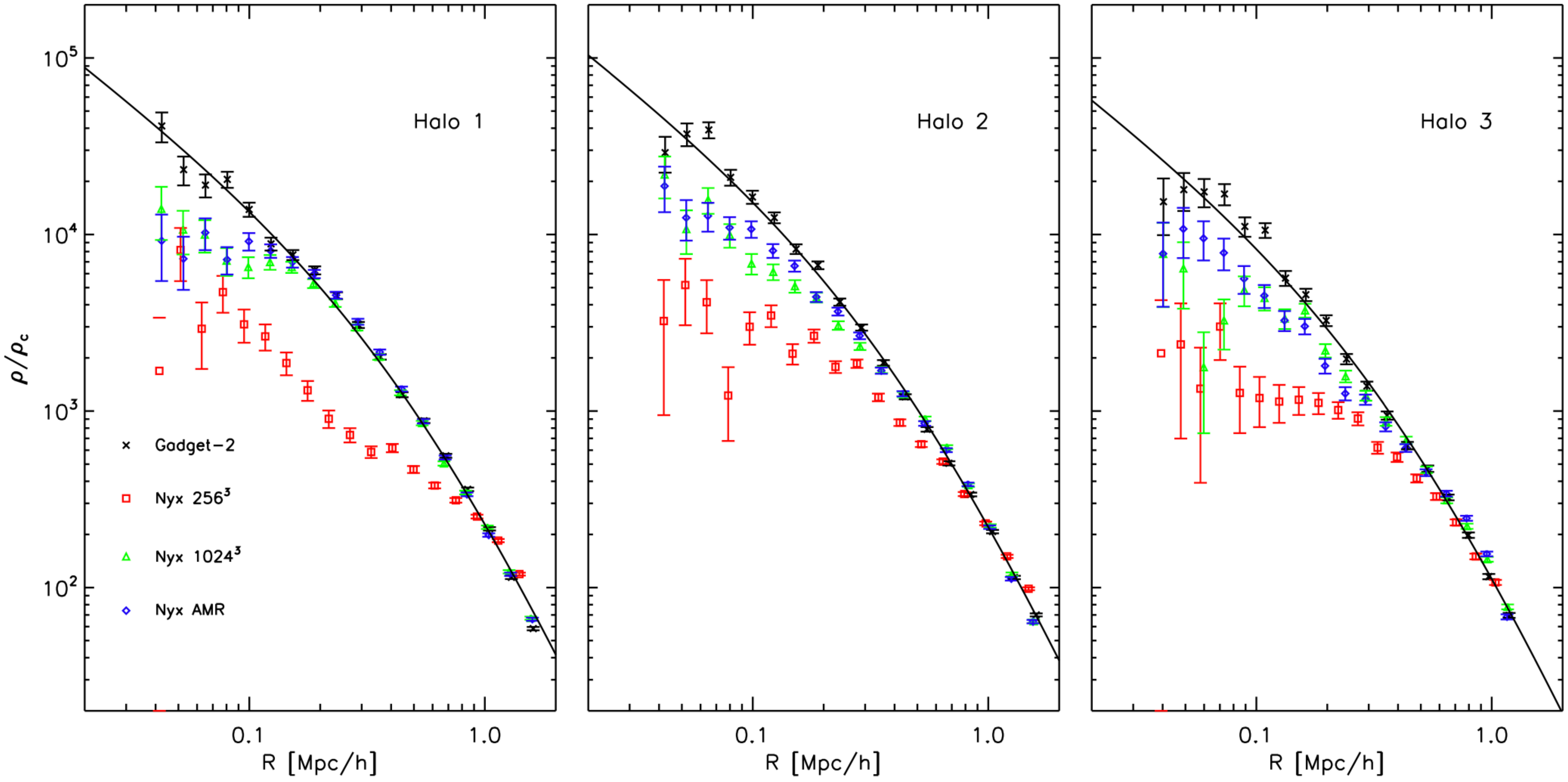
- AMR의 고질적인 문제점 : low mass halo가 억제됨





Validation

Radial Profiles



Thank you!