Galaxy clusters are roughly consisting of dark matter halos (~90%), intra-cluster gases (~9%), and the remaining fraction of compact objects (mainly stars) made out of ordinary matters. In our model, the dark matter halos mainly consists of solitonic cores where each solitonic core may be considered as a ground state of Schrödinger-Newton system.

These solitonic cores have typically masses of about $10^8 M_\odot$ with a size of one kpc scale. Indeed in the simulation of cosmological structure formation with our ultra-light DM, granular structures have been numerically found where each granule may be considered as a solitonic core of the Schrödinger-Newton system.
The Schrödinger-Newton system is described by

\[ i\hbar \partial_t \psi(x, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) + mV(x, t)\psi(x, t) \]

\[ \nabla^2 V(x, t) = 4\pi GM_{\text{tot}} |\psi|^2(x, t) \]

where \( M_{\text{tot}} \) is the total mass of the system and the wave function is normalized by \( \int d^3 x |\psi|^2 = 1 \).
Redefine $\sqrt{M_{\text{tot}}}$ψ → ψ.

The Schrödinger-Newton system then becomes

$$i\hbar \partial_t \psi(x, t) = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) + mV(x, t)\psi(x, t)$$

$$\nabla^2 V(x, t) = 4\pi G |\psi|^2(x, t)$$

where $M_{\text{tot}} = \int d^3x |\psi|^2$.

Hence, this new $|\psi|^2$ can be interpreted as the FDM mass density.

ψ is describing “condensate” of FDM particles moving coherently.
Solitonic ground state and dSphs

- Dwarf spheroidal galaxies (dSph) are relatively small galaxies with spherical shape.
- They typically have mass order of $10^8 M_\odot$ and size of one kpc scale.
More dSphs

- Note almost spherical shape of dwarf spheroidal galaxies.
- They are known to be dark-matter dominated. $R_{DM} > 99\%$!
Ghost galaxy

LMC  Milky Way  Antlia 2
Although at fainter luminosities of dwarf spheroidal galaxies, it is not universally agreed upon how to differentiate between a dwarf spheroidal galaxy and a star cluster; however, many astronomers decide this depending on the object’s dynamics: if it seems to have more dark matter, then it is likely that it is a dwarf spheroidal galaxy rather than a faint star cluster. In the current predominantly accepted Lambda cold dark matter cosmological model, the presence of dark matter is often cited as a reason to classify dwarf spheroidal galaxies as a different class of object from globular clusters, which show little to no signs of dark matter. Because of the extremely large amounts of dark matter in dwarf spheroidal galaxies, they may deserve the title “most dark matter-dominated galaxies.”
The time independent Schrödinger-Newton system then becomes

\[ E_n \psi(x) = -\frac{\hbar^2}{2m} \nabla^2 \psi(x) + mV(x)\psi(x) \]
\[ \nabla^2 V(x) = 4\pi G |\psi|^2(x) \]

with \( M_{tot} = \int d^3x |\psi|^2. \)

Assuming the spherical symmetry \( (\psi(r)) \), the eigenvalue equation can be solved numerically. The ground state corresponds to the soliton core.

All the excite states turn out to be unstable.
Shape of the ground state

DM wave fn. of halo

Halo size
In order to map into the code space, we introduce dimensionless variables by the rescaling

\[ t \rightarrow \tau_c t = \frac{\hbar^3}{m^3 (GM)^2} t \]

\[ \mathbf{x} \rightarrow \ell_c \mathbf{x} = \frac{\hbar^2}{m^2 GM} \mathbf{x} \]

\[ \psi \rightarrow \alpha_\psi \psi = \frac{m^3}{\hbar^3} (GM)^{\frac{3}{2}} \left( \frac{M_{\text{tot}}}{M} \right)^{\frac{1}{2}} \psi \]

\[ V \rightarrow \alpha_V V = \frac{m^2}{\hbar^2} (4\pi GM)^2 V \]

This leads to a form suited for the numerical analysis.
This leads to a form suited for the numerical analysis

\[ i \partial_t \psi(x, t) = -\frac{1}{2} \nabla^2 \psi(x, t) + V(x, t)\psi(x, t) \]

\[ \nabla^2 V(x, t) = 4\pi |\psi|^2(x, t) \]

Note that the normalization of the wave function in the code space becomes \( M_{\text{tot}} = M \int d^3x |\psi|^2 \).

FDM particle mass: \( m = 10^{-22}\text{eV}/c^2 \).

We shall take our reference mass scale as \( M = 4\pi \times 10^7 M_\odot \).

This then fixes the time and length scales as

\[ \tau_c \sim 2.3584 \times 10^7 \text{yr} \]

\[ \ell_c \sim 0.68000 \text{kpc} \]

Hence the unit velocity in the code space corresponds to

\[ v_c = \ell_c/\tau_c \sim 28.194 \text{km/s} \]

in the real space.
We take the soliton mass as $M_s = 4\pi \times 10^7 M_\odot$.

Then the half-mass radius $r_{1/2} = \frac{M}{M_s} \ell_c f_n$ and the energy eigenvalue $E = -\epsilon_n \hbar / \tau_c \left( \frac{M_s}{M} \right)^2$ with

\[
\tau_c \sim 2.3584 \times 10^7 \text{ yr} \\
\ell_c \sim 0.68000 \text{ kpc}
\]

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Shape of the ground state

DM wave fn. of halo

Halo size

Shape of the ground state
Introduce new variables $\rho$ and $S$ such that

$$\psi(x, t) = \rho^{\frac{1}{2}}(x, t)e^{\frac{i}{\hbar}S(x, t)}$$

Then the SN equation can be rewritten as

$$\dot{\rho} + \nabla \cdot \mathbf{v} = 0$$
$$\dot{\mathbf{v}} + v_i \nabla v_i = -\nabla (V + Q)$$

where the velocity is defined by $\mathbf{v} = \frac{1}{m} \nabla S$ and the so-called quantum pressure is given by

$$Q = -\frac{\hbar^2}{2m^2} \rho^{-\frac{1}{2}} \nabla^2 \rho^{\frac{1}{2}}$$

The first is the continuity equation representing conservation of FDM particle number. $V$ is the Newtonian potential and $Q$ is the extra pressure from quantum effect.