Astroparticle Physics

2. The Standard Model of Elementary Paritcles
3. Kinematics and Cross Sections

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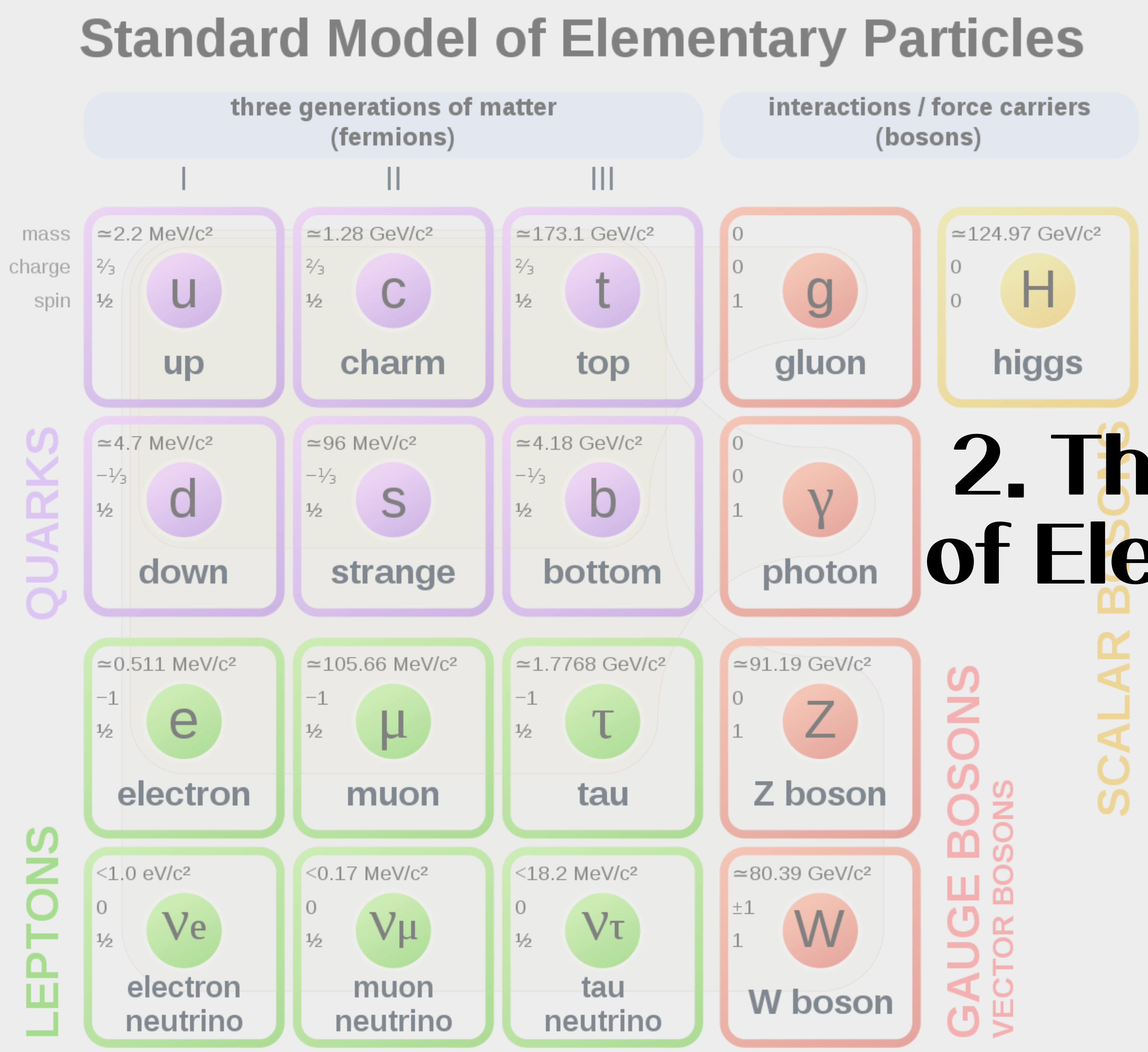
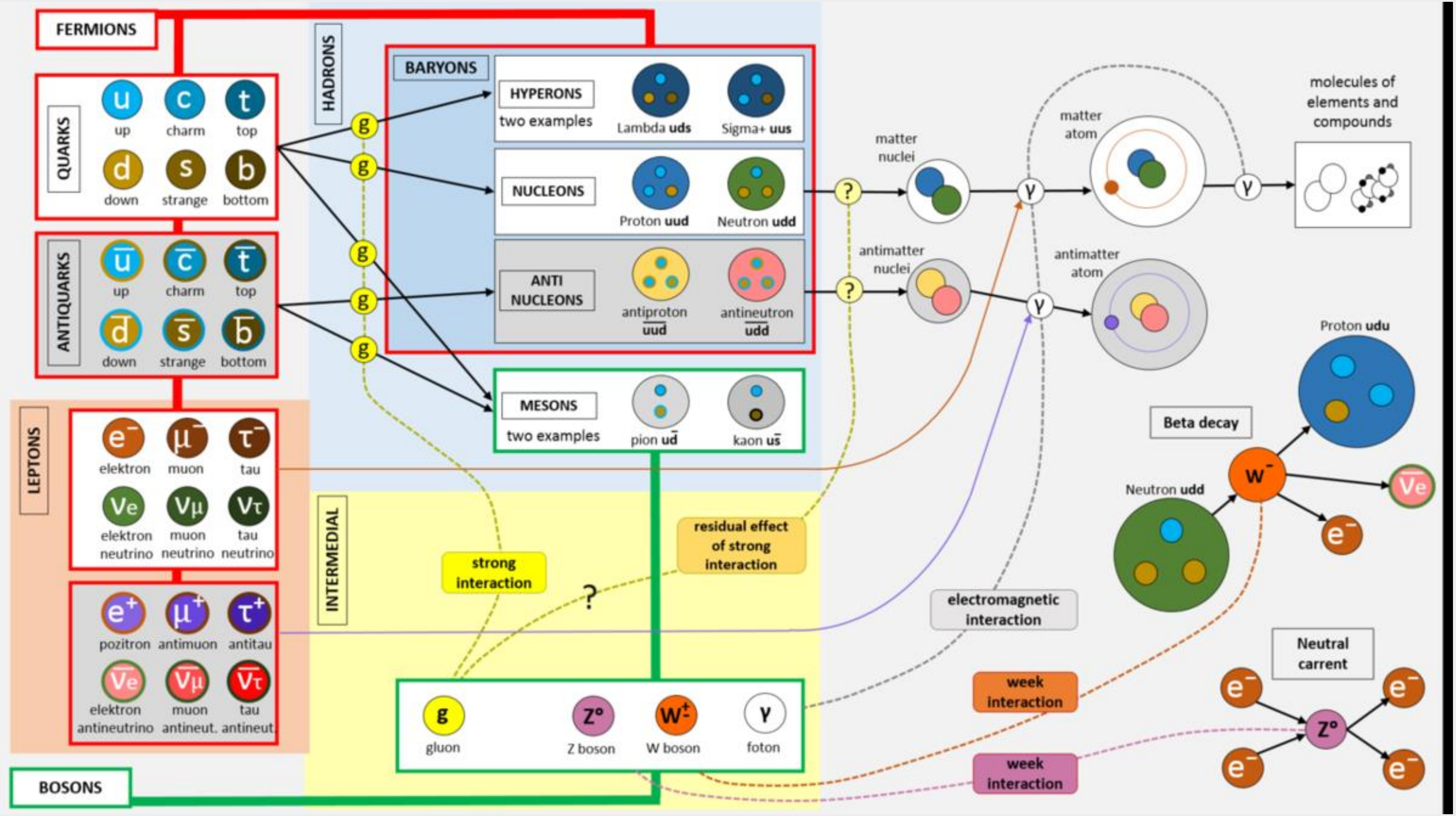
2. The Standard Model of Elementary Particles





FERMIONS
QUARKS
u up
c charm
t top
d down
s strange
b bottom
ANTIQUARKS
up
charm
top
down
strange
bottom
LEPTONS
e⁻ elektron
μ⁻ muon
τ⁻ tau
Ve elektron neutrino
Vμ muon neutrino
Vτ tau neutrino
e⁺ pozitron
μ⁺ antimuon
τ⁺ antitau
elektron antineutrino
muon antineut.
tau antineut.
BOSONS
HADRONS
BARYONS
HYPERONS
two examples
Lambda uds
Sigma+ uus
NUCLEONS
Proton uud
Neutron udd
ANTI NUCLEONS
antiproton uud
antineutron udd
MESONS
two examples
pion ud
kaon us
INTERMEDIAL
strong interaction
residual effect of strong interaction
gluon
Z boson
W boson
foton
matter nuclei
antimatter nuclei
matter atom
antimatter atom
molecules of elements and compounds
electromagnetic interaction
Beta decay
Proton udu
Neutron udd
week interaction
week interaction
Neutral carrent

elementary particle

made of two or more quarks and held together by the strong force.
And depending on the **number of baryon** this is divided into two types : **baryon, meson**

Hadron(강입자)

Fermion

Boson

Baryon(중입자)

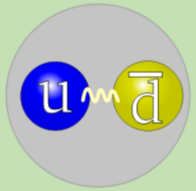
of baryon = 1

This is composed by three quarks.
: proton(양성자), neutron(중성자)

Meson(중간자)

of baryon = 0

This is composed by a quark and a antiquark
: pion(파이온), kaon(케이온)



- What is fermion?

Fermion has half odd integer spin: spin ½, spin 3/2, etc…

- What is boson?

boson has integer spin: spin 0, spin1, spin 2, etc…

A fundamental constituent of matter

Quark(쿼크)

Quark compose **baryon(중입자)** and **meson(중간자).**
quarks are never found in isolation because of **quark confinement**

Classification of matter			
Type	Generations of matter		
	First	Second	Third
Quarks			
up-type	up	charm	top
down-type	down	strange	bottom
Leptons			
charged	electron	muon	tau
neutral	electron neutrino	muon neutrino	tau neutrino

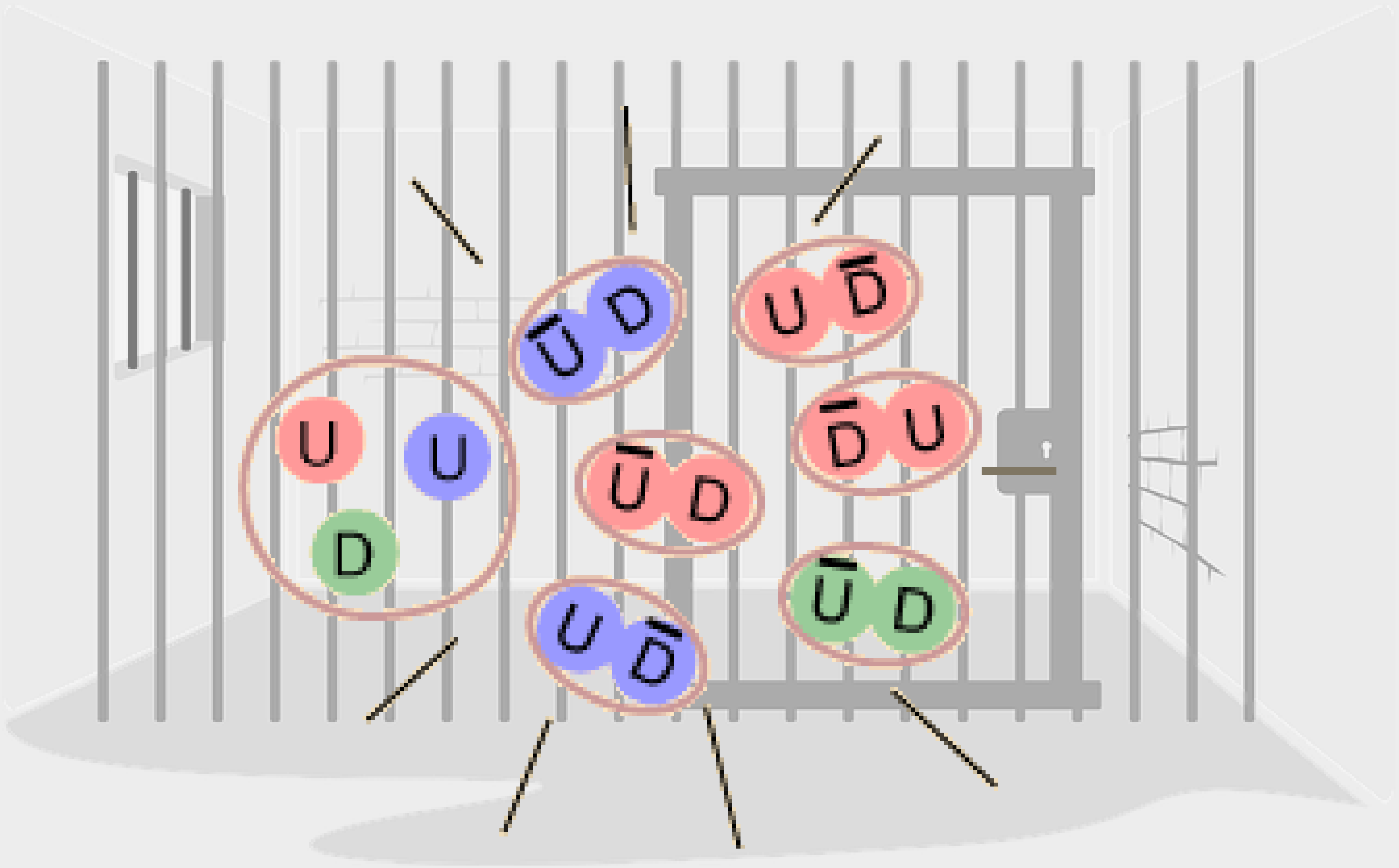
Lepton(경입자)

Lepton exist freely.
Depending on charge, this is divided into two types : electron-like leptons, neutrinos

- ❖ Quarks and leptons appear to be pointlike particles, having no internal structure.
- ❖ Quarks and leptons are fermion

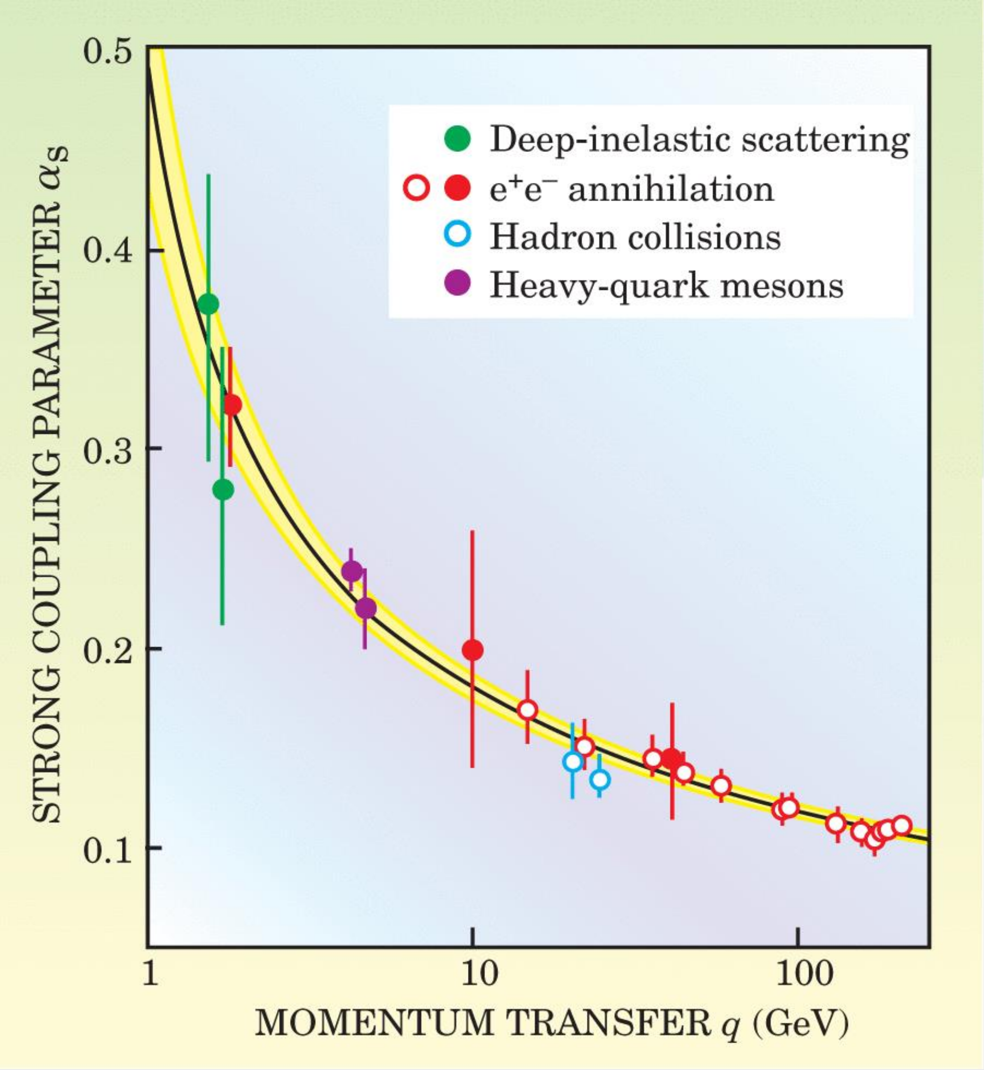
Quark Confinement(쿼크 가둠)

- ❖ While atoms, protons and neutrons can be observed as free particles in experiments, quarks can never escape from their hadronic prison.
- ❖ **Nobody has ever been able to find free quark**



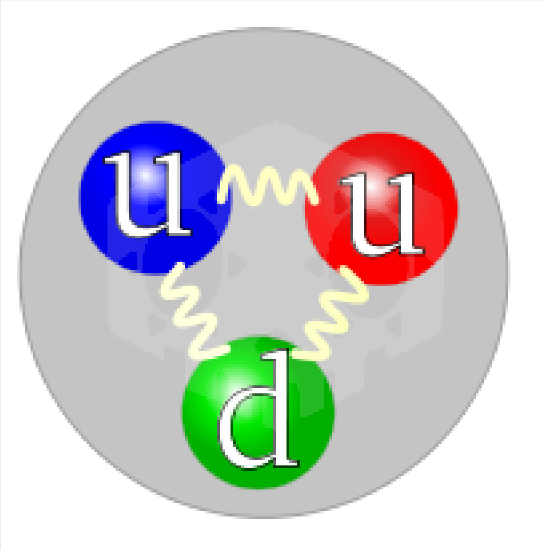
QCD

❖ Quantum Chromodynamics(= describes the interaction of quarks) explains the **asymptotic freedom(점근 자유성)** of quarks at high momenta.



-> Bound quarks(=hadron) typically have low momenta and are subjected to 'infrared slavery'

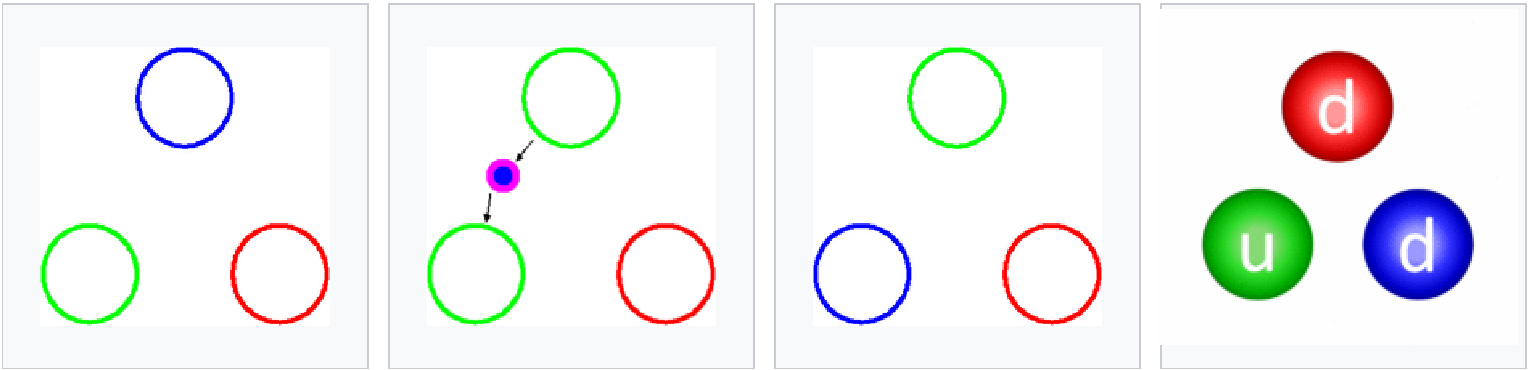
-> This confinement does not allow the quarks to separate from each other.



Colour Confinement(색 가둠)

◆ We just assume that quarks have colours(red, green, blue).
And also antiquarks have colours(anti-red, anti-green, anti-blue)

◆ The quarks that form hadrons are held together by the exchange of gluons.
since gluons mediate the interactions between quarks so they must possess two colours: colour-anticolour



A hadron with 3 quarks (red, green, blue) before a color change

Blue quark emits a blue-antigreen gluon

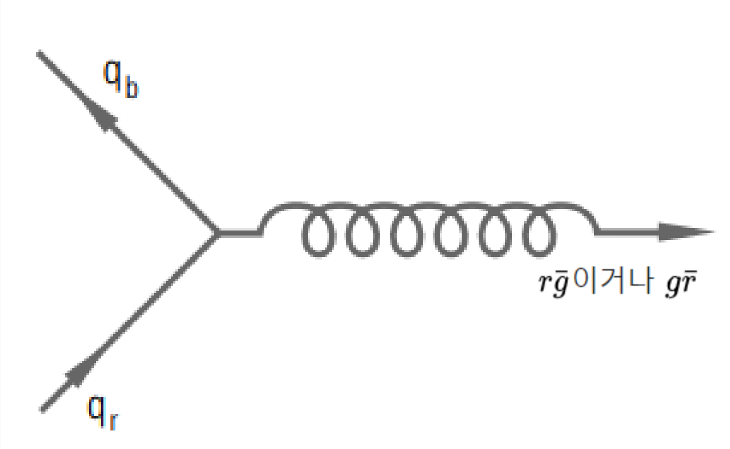
Green quark has absorbed the blue-antigreen gluon and is now blue; color remains conserved

An animation of the interaction inside a neutron. The gluons are represented as circles with the color charge in the center and the anti-color charge on the outside.

◆ When the three colours combined, it become colorless.

◆ quarks only exist with colorless and that’s why we cannot find isolated quarks.

	r	g	b
$-r$	$r\bar{r}$	$g\bar{r}$	$b\bar{r}$
$-g$	$r\bar{g}$	$g\bar{g}$	$b\bar{g}$
$-b$	$r\bar{b}$	$g\bar{b}$	$b\bar{b}$



Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

There are 8 types of gluons not 9

$$(r\bar{b} + b\bar{r})/\sqrt{2} \qquad (r\bar{g} + g\bar{r})/\sqrt{2} \qquad (b\bar{g} + g\bar{b})/\sqrt{2}$$

$$-i(r\bar{b} - b\bar{r})/\sqrt{2} \qquad -i(r\bar{g} - g\bar{r})/\sqrt{2} \qquad -i(b\bar{g} - g\bar{b})/\sqrt{2}$$

$$(r\bar{r} - b\bar{b})/\sqrt{2} \qquad (r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$$

intermediate particle

A particle which mediates a force between two particles

W and Z bosons

These elementary particles mediate the **weak interaction**; the respective symbols are W^+, W^-, and Z^0.

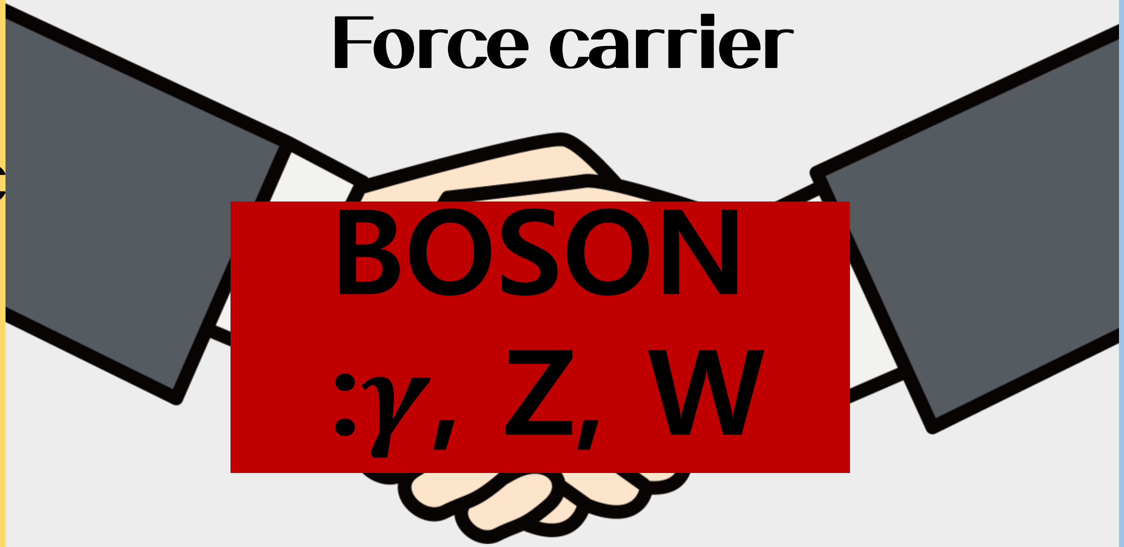
Electric charge : W^+ = 1, W^- = -1 , Z^0 = 0
The three particles have a spin of 1.(this is why they are called ‘boson’)

Why they have different name?

The W bosons are named after the ***weak*** force
The W bosons had already been named, and Z bosons appeared later.
The Z bosons were named for having ***zero*** electric charge

Electromagnetic force

Force carrier



BOSON

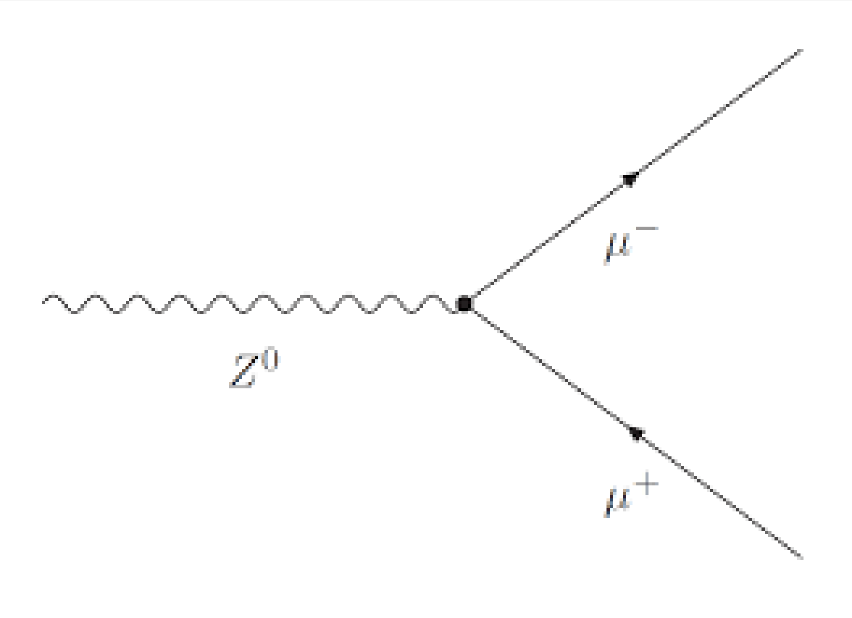
:γ, Z, W

Weak interaction

W and Z decay

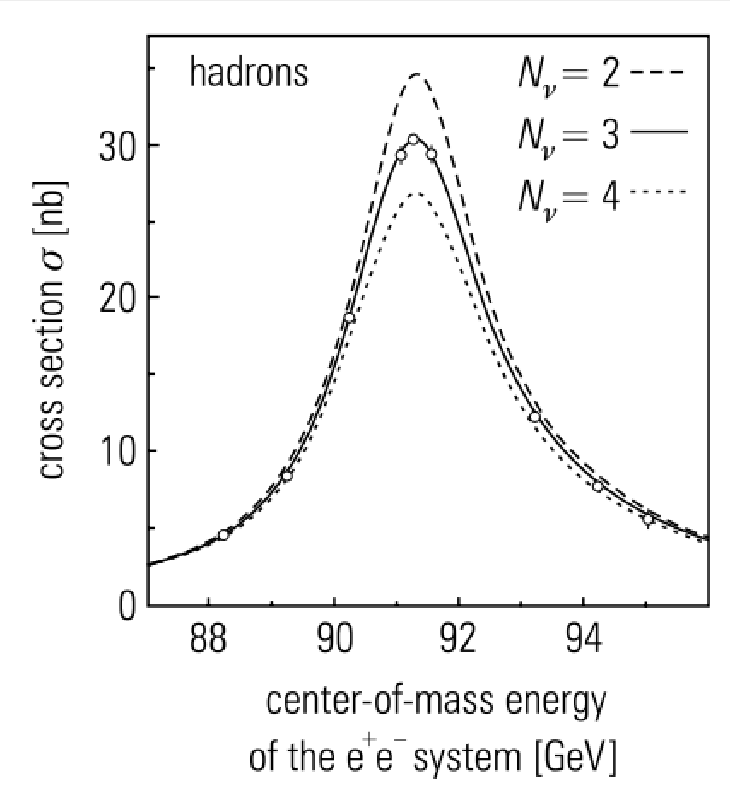
The W and Z bosons decay to fermion pairs

- **W bosons** can decay to a lepton and anti-lepton or to a quark and antiquark
- **Z bosons** decay into a fermion and its antiparticle.



Through this,
we can know about existence of three charged leptons
: electrons, muons, tau.

This result was obtained from the measurement of the total **Z decay width**.



decay width Γ is measured in units of energy

From the Heisenberg's uncertainty principle,

$$\Delta E \ \Delta t \geq \hbar/2 \ \ (\hbar = h/2\pi)$$

$\Delta t = \tau$: the lifetime of the particle

$\Delta E = \Gamma$: decay width

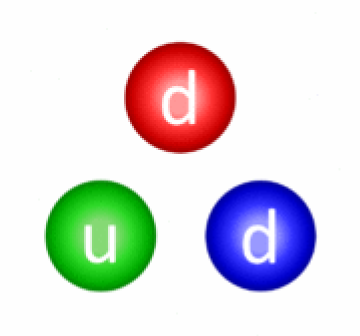
when τ is shorter, the Γ is larger

A large number(not 1) of different particles will reduce Z life time(τ)
then the decay width(Γ) will increasing

electroweak interaction	γ	W^-	W^+	Z
spin [$\hbar$]	1	1	1	1
electric charge [e]	0	-1	$+1$	0
mass [GeV/c^2]	0	80.4	80.4	91.2

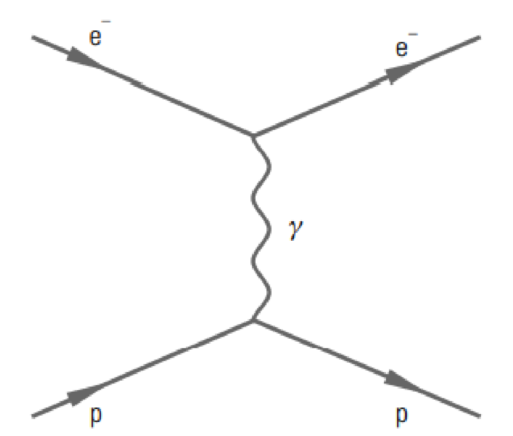
strong interaction	gluon g
spin [$\hbar$]	1
electric charge [e]	0
mass [GeV/c^2]	0

gravitational interaction	graviton G
spin [$\hbar$]	2
electric charge [e]	0
mass [GeV/c^2]	0



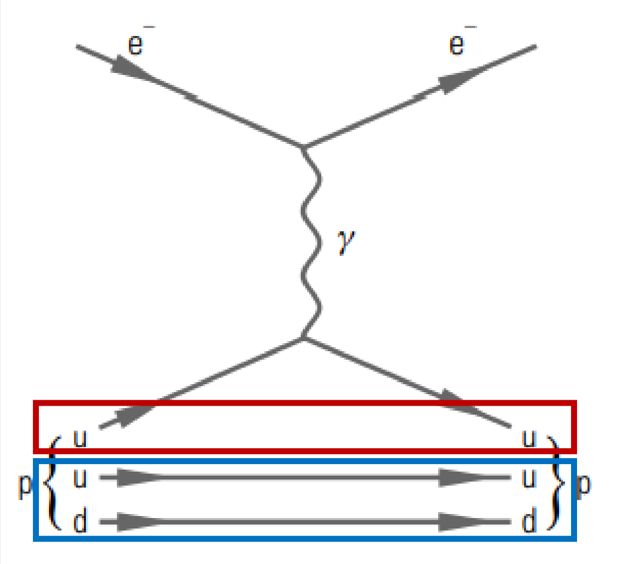
Feynman diagrams

Which represent interactions of elementary particles



Rutherford scattering of electrons on protons

high energies

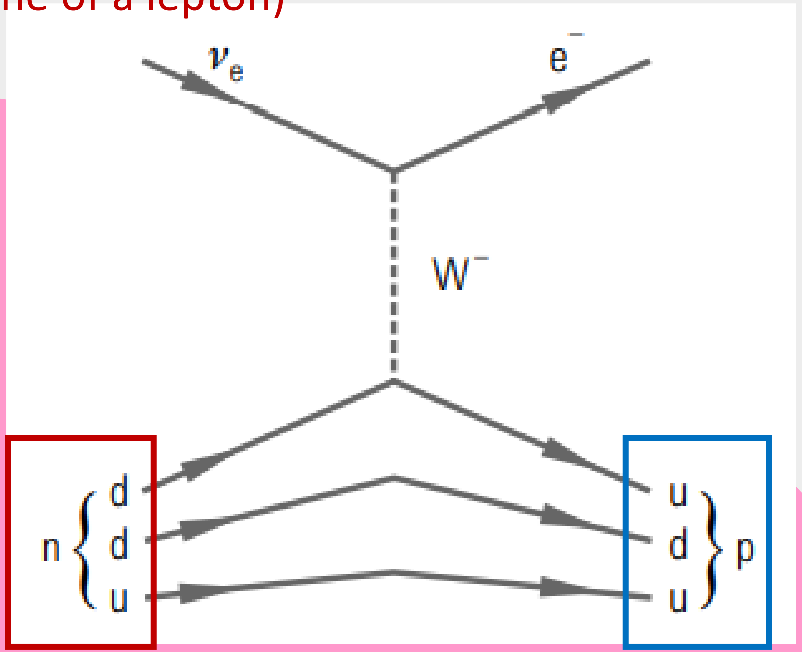


❖ At high energies the photon does not interact with proton as a whole, but rather only with one of its constituent quarks.
The other quarks is only as spectators.

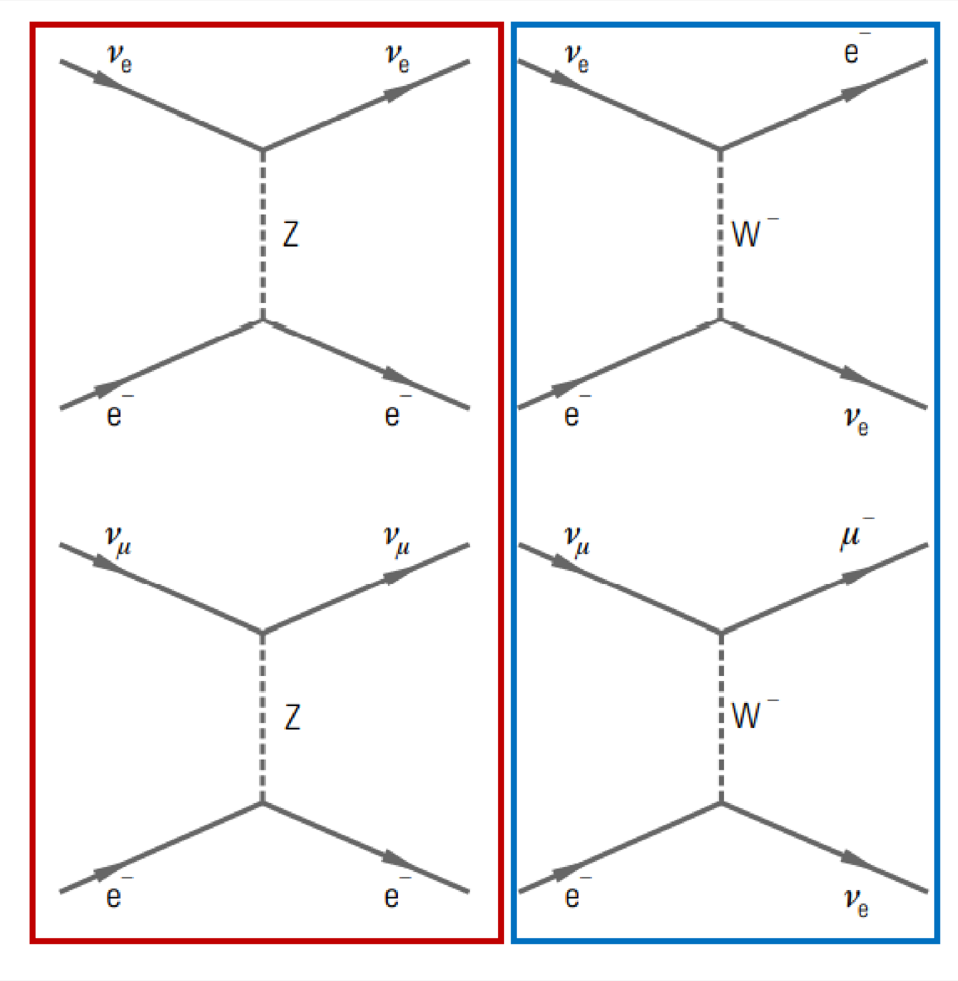
❖ And photons cannot change the nature of a target particle in an interaction

In weak interaction

Electron neutrino
(one of a lepton)

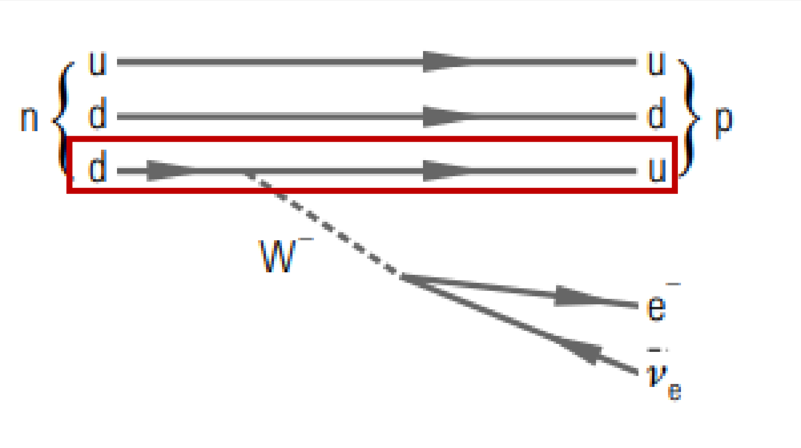


But in weak interactions(which occur due to the exchange of W and Z bosons), the bosons can cause an interchange between particles.



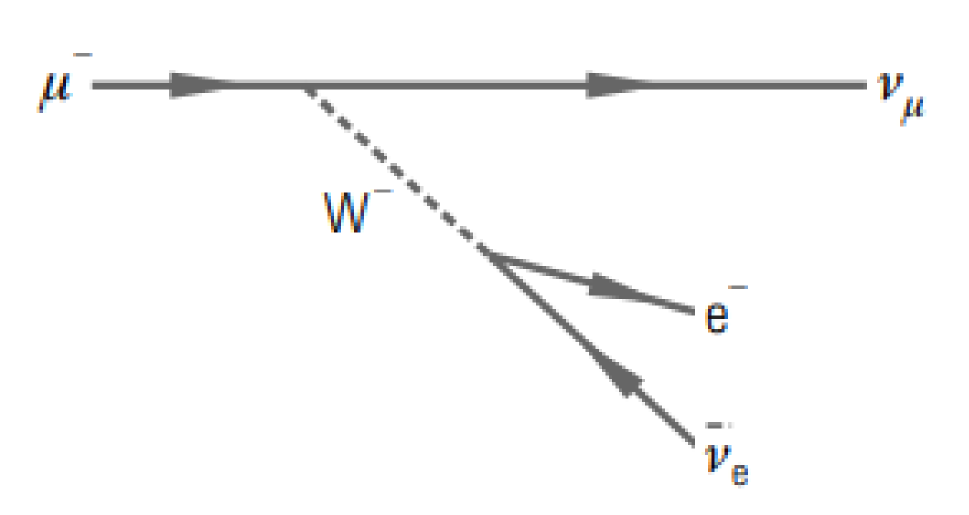
- Z exchange(neutral-current interaction) can not change the particles, But charged neutrino(W) can change the particles.

Neutron decay



down quark in the neutron is transformed into a up quark by the emission of a W−.
The W− immediately decays into a lepton(W− → e− and $\overline{\nu_e}$).

Muon decay



Quantum number

The various elementary particles are characterized by quantum numbers.

Quantum Number	Symbol	Possible values
Principal Quantum Number	n	4
Azimuthal Quantum Number	l	2
Magnetic Quantum Number	m_l	$-2,-1,0,1,2$
Spin Quantum Number	m_s	$+\frac{1}{2},-\frac{1}{2}$

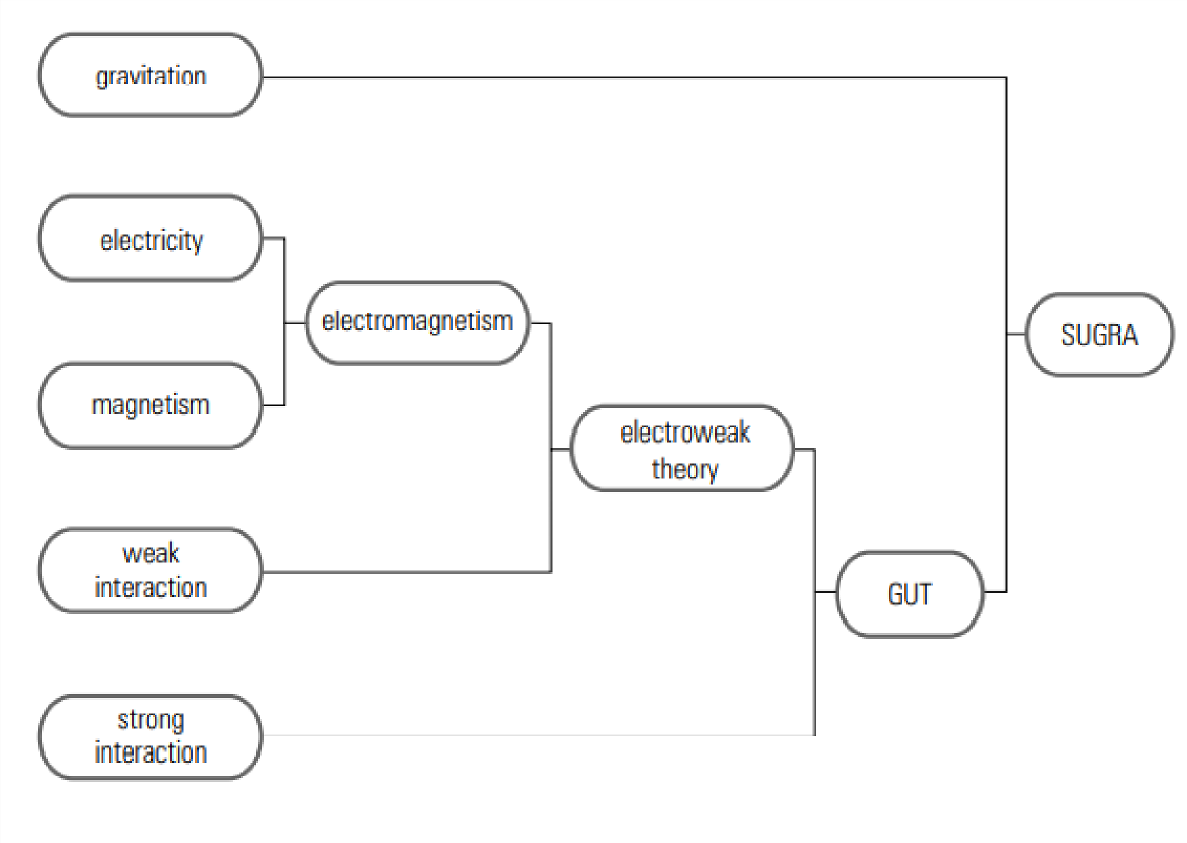
Conservation laws of particle physics

physical quantity	interaction		
	strong	electromagnetic	weak
momentum	+	+	+
energy (incl. mass)	+	+	+
ang. momentum	+	+	+
electric charge	+	+	+
quark flavour	+	+	–
lepton number*	./.	+	+
parity	+	+	–
charge conjugation	+	+	–
strangeness	+	+	–
isospin	+	–	–
baryon number	+	+	+

The lepton number is not relevant for strong interactions

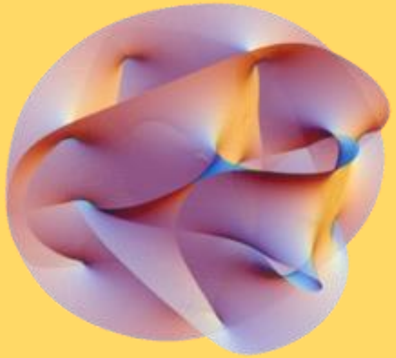
Theory of everything

There have been many attempts to formulate a Theory of everything that unites all interactions.
A very promising candidate for such a global description is string theory.



In the text book, the all-embracing theory of supergravity(SUGRA) is embedded in the M theory = an 11-Dimensional superstring theory.

The 'M' stands for membrane, Matrix, mystery, or mother



3. Kinematics and cross sections

Relativistic Kinematics(상대 운동학)

$$E = mc^2$$

m = mass of a particle
v = velocity
c = velocity of light in vacuum

❖ Particles with velocity near to c do not get faster when accelerated because of relativistic mass increase.

relativistic mass increase

$$m = \frac{m_0}{\sqrt{1-\beta^2}} = \gamma m_0$$

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

m_0 = rest mass
β = v/c (particle velocity normalized by c)
m_0c^2 = rest energy of particle

Energy

$$E = \gamma m_0 c^2$$

momentum

$$p = mv = \gamma m_0 \beta c$$

❖ Energy Difference between E(velocity = v) and E(velocity = c)

$$E^2 - p^2c^2 = \gamma^2 m_0^2 c^4 - \gamma^2 m_0^2 \beta^2 c^4$$

β = v/c = c/c = 1

$$= \frac{m_0^2 c^4}{1-\beta^2}(1-\beta^2) = m_0^2 c^4$$

Lorentz-invariant quantity
: this quantity is the same in all systems
and it equals the square of the rest energy

❖ Using this, the total energy of a relativistic particle can be expressed by this:

$$E = c\sqrt{p^2 + m_0^2 c^2}$$

This equation can apply for all particles even m_0 = 0

$$E = c\sqrt{p^2 + m_0^2c^2}$$ → From this equation we can get kinetic energy

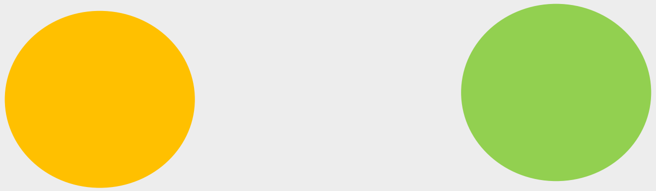
Total energy rest energy

$$\begin{aligned} E^{\text{kin}} &= E - m_0c^2 = c\sqrt{p^2 + m_0^2c^2} - m_0c^2 \\ &= m_0c^2\sqrt{1 + \left(\frac{p}{m_0c}\right)^2} - m_0c^2 \\ &\approx m_0c^2\left(1 + \frac{1}{2}\left(\frac{p}{m_0c}\right)^2\right) - m_0c^2 \\ &= \frac{p^2}{2m_0} = \frac{1}{2}m_0v^2\ , \end{aligned}$$

Threshold energy

- In particle physics, the **threshold energy** can be called **minimum kinetic energy** for production of a particle
- The **threshold energy** is always greater than or equal to the **rest energy** of the desired particle.

for example, through the electron–positron(전자 – 양전자) head-on collision, we want to create a particle of mass M. electron and positron have the same total energy E and this should satisfy this: 2E≥ M .



Total energy = E_1
Momenta = p_1

Total energy = E_2
Momenta = p_2

θ is the angle between p1 and p2

center-of-mass energy E_{CMS}

$$E_{\mathrm{CMS}} = \sqrt{s}$$
$$= \left\{(E_1 + E_2)^2 - (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2\right\}^{1/2}$$
$$= \left\{E_1^2 - p_1^2 + E_2^2 - p_2^2 + 2E_1E_2 - 2\boldsymbol{p}_1 \cdot \boldsymbol{p}_2\right\}^{1/2}$$
$$= \left\{m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta)\right\}^{1/2}$$

$$
\begin{aligned}
E_{\mathrm{CMS}} &= \sqrt{s} \\
&= \left\{(E_1 + E_2)^2 - (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2\right\}^{1/2} \\
&= \left\{E_1^2 - p_1^2 + E_2^2 - p_2^2 + 2E_1E_2 - 2\boldsymbol{p}_1 \cdot \boldsymbol{p}_2\right\}^{1/2} \\
&= \left\{m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta)\right\}^{1/2}
\end{aligned}
$$

(β1, β2 → 1 and m1, m2 << E1, E2 and angles θ is not too small)

$$E_{\mathrm{CMS}} = \sqrt{s} \approx \{2E_1E_2(1 - \cos\theta)\}^{1/2}$$

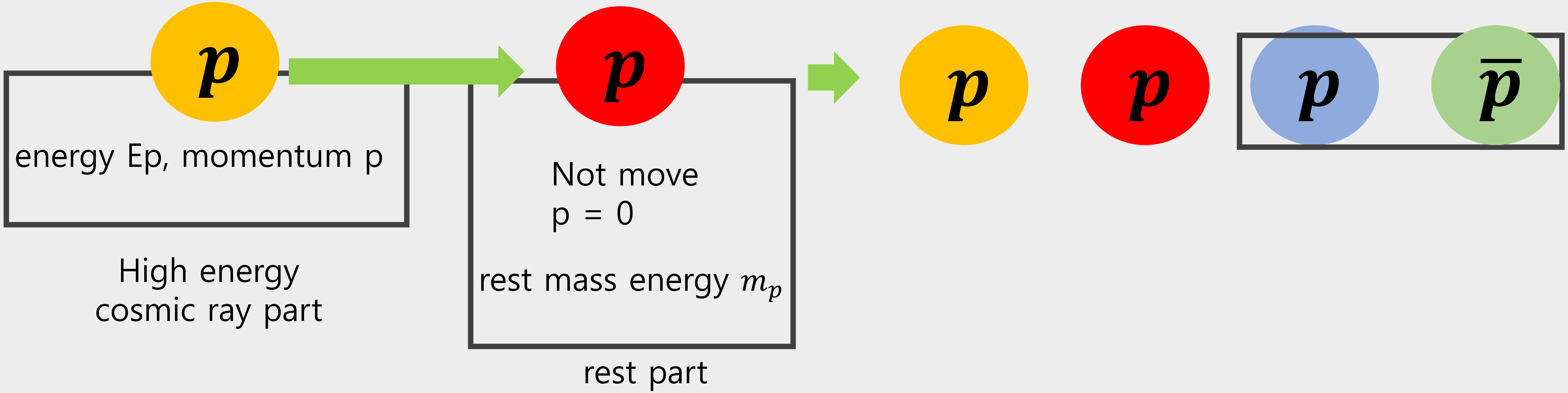
→ This can be threshold energy

Example

Let's solve threshold energy and kinetic energy of the $\bar{p}p$ production

assume that a high-energy cosmic-ray proton produces a proton–antiproton pair on a target proton at rest.

(energy Ep, momentum p, rest mass m_p)

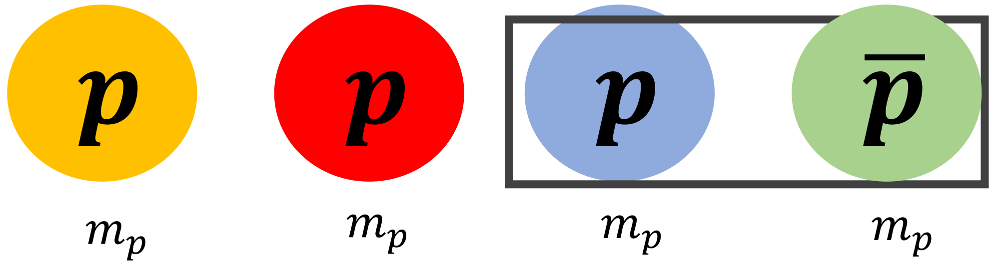


center-of-mass energy E_{CMS}

$$
\begin{aligned}
E_{\mathrm{CMS}} &= \sqrt{s} \\
&= \left\{(E_1 + E_2)^2 - (\boldsymbol{p}_1 + \boldsymbol{p}_2)^2\right\}^{1/2} \\
&= \left\{E_1^2 - p_1^2 + E_2^2 - p_2^2 + 2E_1E_2 - 2\boldsymbol{p}_1 \cdot \boldsymbol{p}_2\right\}^{1/2} \\
&= \left\{m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta)\right\}^{1/2}
\end{aligned}
$$

Apply our situation

$$
\begin{aligned}
\sqrt{s} &= \left\{(E_p + m_p)^2 - (\boldsymbol{p} - 0)^2\right\}^{1/2} \\
&= \left\{E_p^2 + 2m_pE_p + m_p^2 - p^2\right\}^{1/2} \\
&= \left\{2m_pE_p + 2m_p^2\right\}^{1/2} .
\end{aligned}
$$



$$\sqrt{s} \geq 4m_p$$

$$
\begin{aligned}
2m_pE_p + 2m_p^2 &\geq 16\,m_p^2 , \\
E_p &\geq 7m_p \;(= 6.568\ \mathrm{GeV})
\end{aligned}
$$

threshold energy

$$E_p^{\mathrm{kin}} = E_p - m_p \geq 6m_p$$

kinetic energy minimum

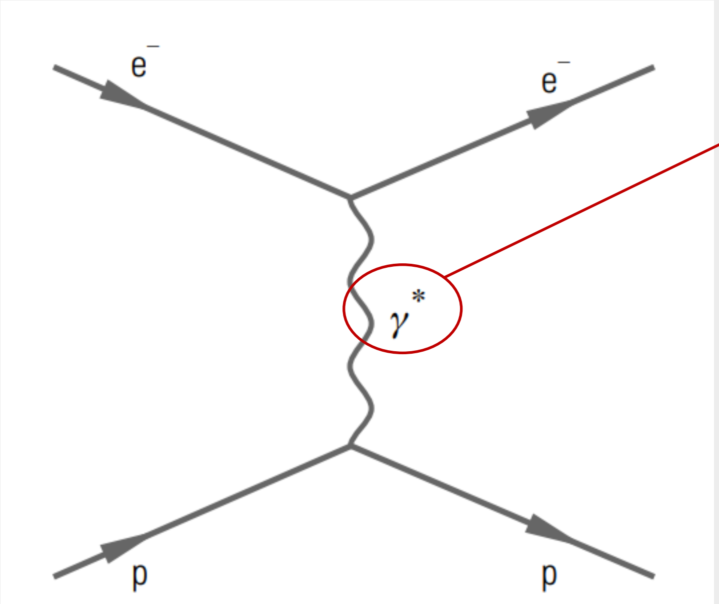
Four vectors

By combining position vector (x,y,z) and time t, we can made four vectors

And four-momentum vector looks like this: $q = \begin{pmatrix} E \\ \boldsymbol{p} \end{pmatrix}$ with $\boldsymbol{p} = (p_x, p_y, p_z)$

Example

Electron–proton scattering



When particles collide,
Real particles can borrow energy for a short time from the vacuum within the framework of Heisenberg's uncertainty principle.
Such particles are called virtual.
virtual particles can only occur as exchange particles.

Let's calculate this virtual particle's mass!

Incoming electron

$$q_e = \begin{pmatrix} E_e \\ p_e \end{pmatrix}$$

Incoming protron

$$q_p = \begin{pmatrix} E_p \\ p_p \end{pmatrix}$$

Final state electron

$$q_e{}' = \begin{pmatrix} E'_e \\ p'_e \end{pmatrix}$$

Final state protron

$$q_p{}' = \begin{pmatrix} E'_p \\ p'_p \end{pmatrix}$$

four-momentum conservation holds:

$$q_e + q_p = q_e{}' + q_p{}'$$

$$\begin{aligned} q_{\gamma*}^2 &= (q_e - q_e')^2 \\ &= \begin{pmatrix} E_e - E_e' \\ \boldsymbol{p}_e - \boldsymbol{p}_e' \end{pmatrix}^2 = (E_e - E_e')^2 - (\boldsymbol{p}_e - \boldsymbol{p}_e')^2 \\ &= E_e^2 - \boldsymbol{p}_e^2 + E_e'^2 - \boldsymbol{p}_e'^2 - 2E_eE_e' + 2\boldsymbol{p}_e \cdot \boldsymbol{p}_e' \\ &= 2m_e^2 - 2E_eE_e'(1 - \beta_e\beta_e' \cos\theta) \end{aligned}$$

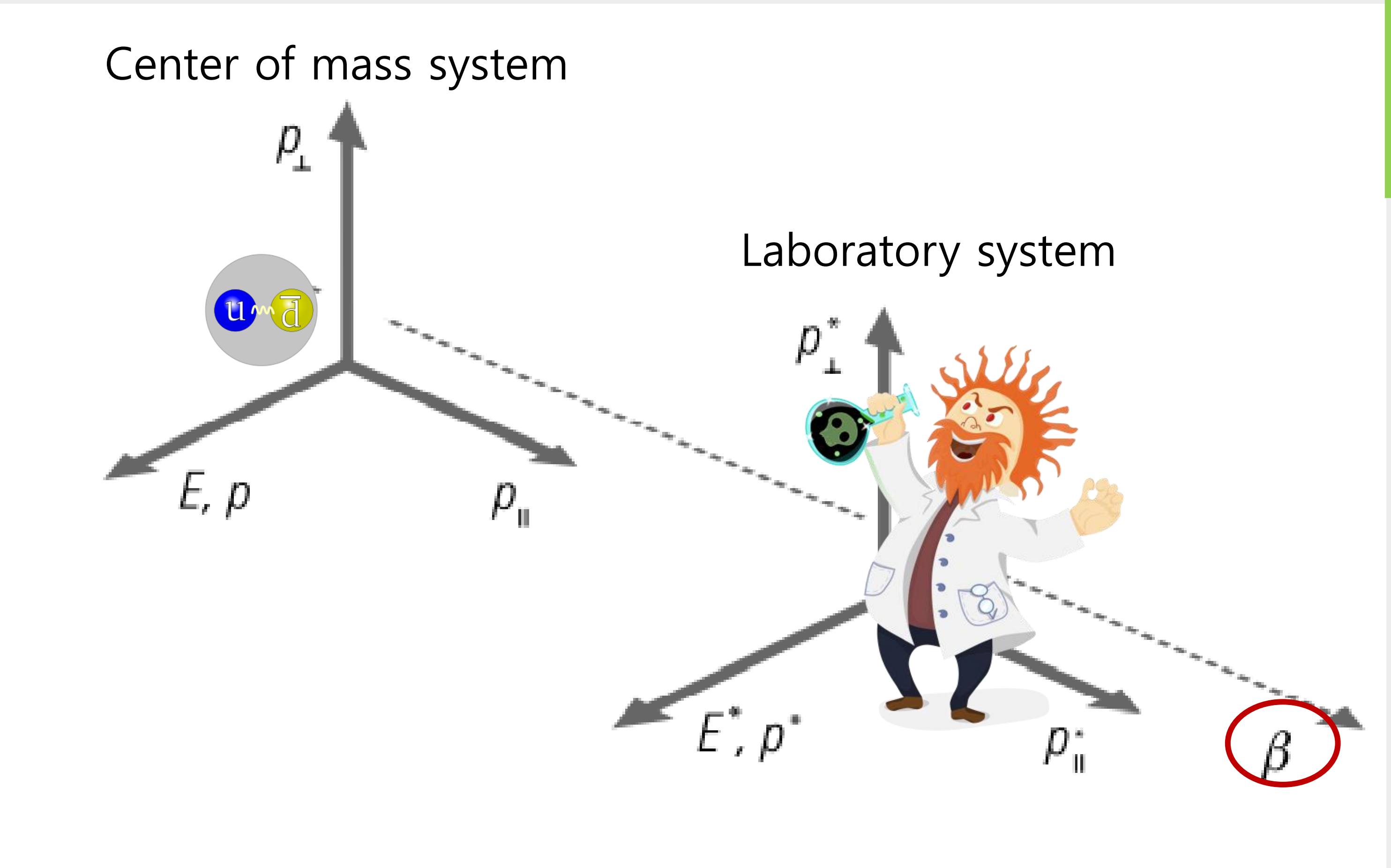
$$\begin{aligned} q_{\gamma*}^2 &= -2E_eE_e'(1 - \cos\theta) \\ &= -4E_eE_e' \sin^2\frac{\theta}{2} \,. \end{aligned}$$

$$q_{\gamma*}^2 = -E_eE_e'\theta^2$$

- in this case, the four-momentum squared of the exchanged virtual photon is negative

- This means that the mass of γ ∗ is purely imaginary.
- Such photons are called *space-like*

Lorentz Transformation



Laboratory system — Center of mass system

$$\begin{pmatrix} E^* \\ p_\parallel^* \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}, \quad p_\perp^* = p_\perp$$

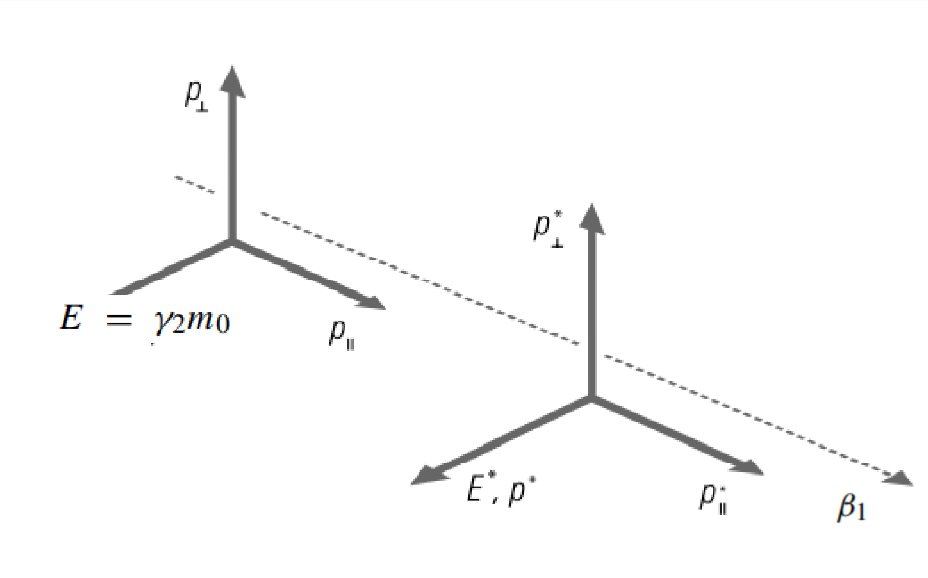
$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = v/c$$

*Laboratory system이 Center of mass system 에 대해 정지하고 있을 때

When $\beta = 0$, $\gamma = 1$

$E* = E$ and $p* = p$.

Relationship between energy and rest mass



$$\begin{pmatrix} E^* \\ p^*_\parallel \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}$$

$$E^* = \gamma E - \gamma\beta p_\parallel$$

$$p^*_\parallel = -\gamma\beta E + \gamma p_\parallel$$

Apply our situation

$$\begin{aligned} E^* &= \gamma_1 E - \gamma_1 \beta_1 p_\parallel \\ &= \gamma_1 \gamma_2 m_0 - \gamma_1 \frac{\sqrt{\gamma_1^2 - 1}}{\gamma_1} \sqrt{(\gamma_2 m_0)^2 - m_0^2} \\ &= \gamma_1 \gamma_2 m_0 - m_0 \sqrt{\gamma_1^2 - 1} \sqrt{\gamma_2^2 - 1} \end{aligned}$$

a Laboratory system that moves along with a particle(Center of mass system)

*Laboratory system이 Center of mass system과
같은 속도로 움직이고 있을 때

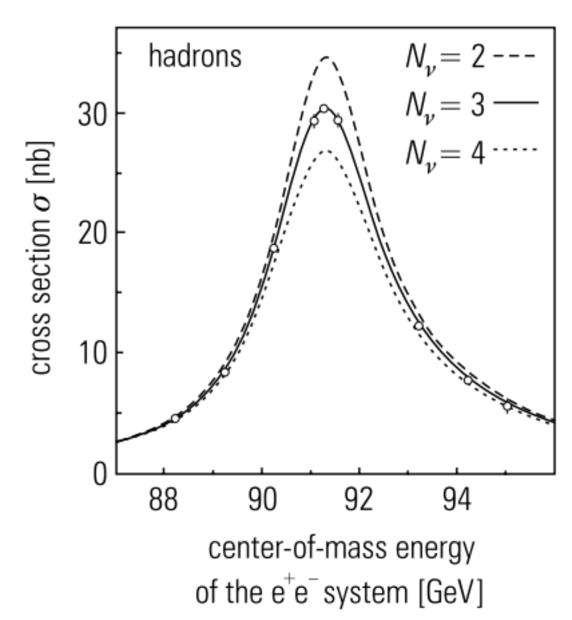
$$\gamma_1 = \gamma_2 = \gamma$$

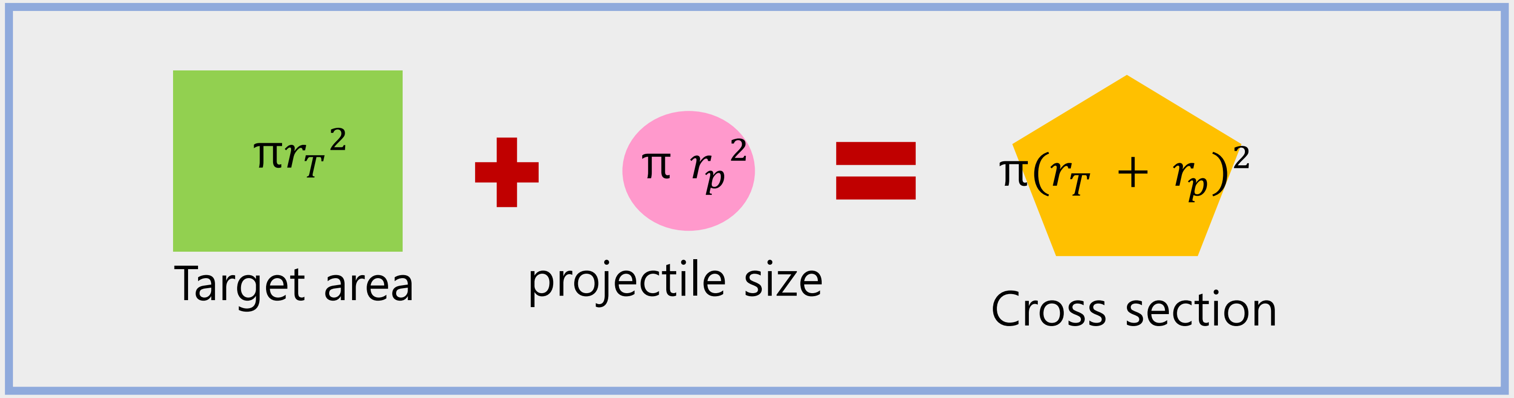
$$E^* = \gamma^2 m_0 - m_0(\gamma^2 - 1) = m_0$$

Cross Sections

- In the most simple case the cross section can be considered as an effective area which the target particle represents for the collision with a projectile

- If the target has an area of $\pi {r_T}^2$ and the projectile size corresponds to $\pi\ {r_p}^2$
  Cross section is $\pi(r_T\ +\ r_p)^2$





In most cases the cross section also depends on other parameters.
The atomic cross section σ_A which is measured in cm^2, is related to the interaction length λ

$$\lambda\,\{\mathrm{cm}\} = \frac{A}{N_\mathrm{A}\,\{\mathrm{g}^{-1}\}\,\varrho\,\{\mathrm{g/cm^3}\}\,\sigma_\mathrm{A}\,\{\mathrm{cm^2}\}}$$

$$\mu\,\{\mathrm{cm}^{-1}\} = \frac{N_\mathrm{A}\,\varrho\,\sigma_\mathrm{A}}{A} = \frac{1}{\lambda}$$

$$\phi\,\{(\mathrm{g/cm^2})^{-1}\} = \frac{\mu}{\varrho} = \frac{N_\mathrm{A}}{A}\,\sigma_\mathrm{A}$$