How to detect ULDM

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Outline

1. ULDM (Fuzzy DM)

2. Gravitational detection of FDM

3. non-gravitational detection and SIULDM

Challenges for ΛCDM

•ΛCDM was very successful but is slowly becoming non-standard?

cf) 2105.05208

1. Small scale crisis (on galactic scale and below) :
   predicts too many small structures not observed

2. Hubble tension ~ 5σ:
   H_0 mismatch between CMB and SN

3. S_8 tension ~ 2-3σ : $S_8 \equiv \sigma_8\sqrt{\Omega_m/0.3}$, z < 2, Mpc
   matter density fluctuation amplitude mismatch
   between CMB and WL & Cluster

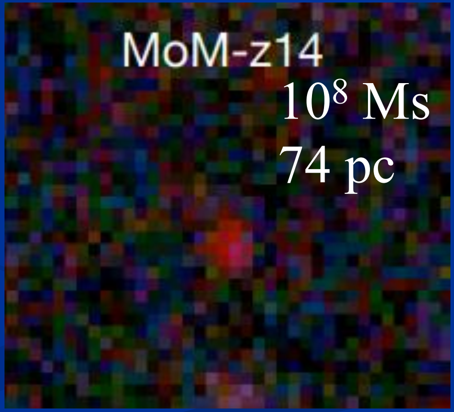
4. DE is not Λ? ~ 4σ (ex, AP test, DESI, SN?)

5. Too early SMBHs (z~11) and galaxies (z~14, 32?)

6. Galaxy cluster collision (collision speed & DM-star offset)

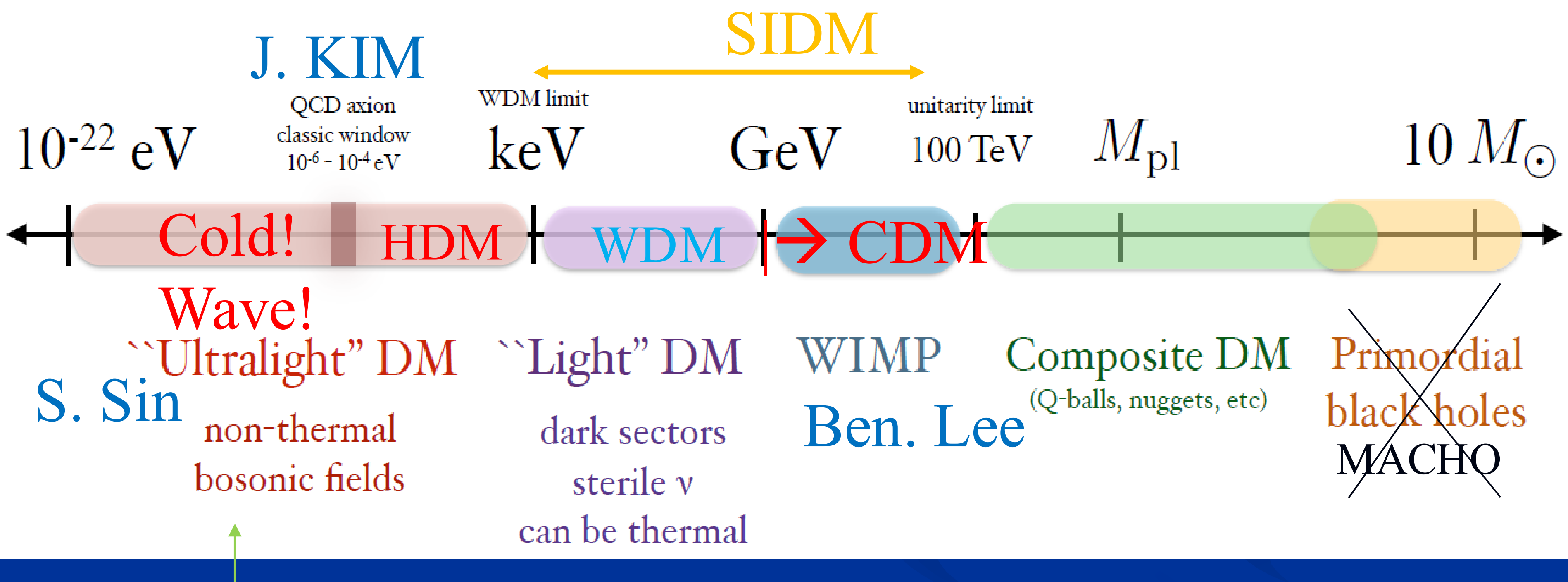
7. Li problem, Cosmic birefringence

…etc → Any good DM model should address these tensions



Mass scale of dark matter

(not to scale) TASI lectures by Lin arXiv:1904.07915

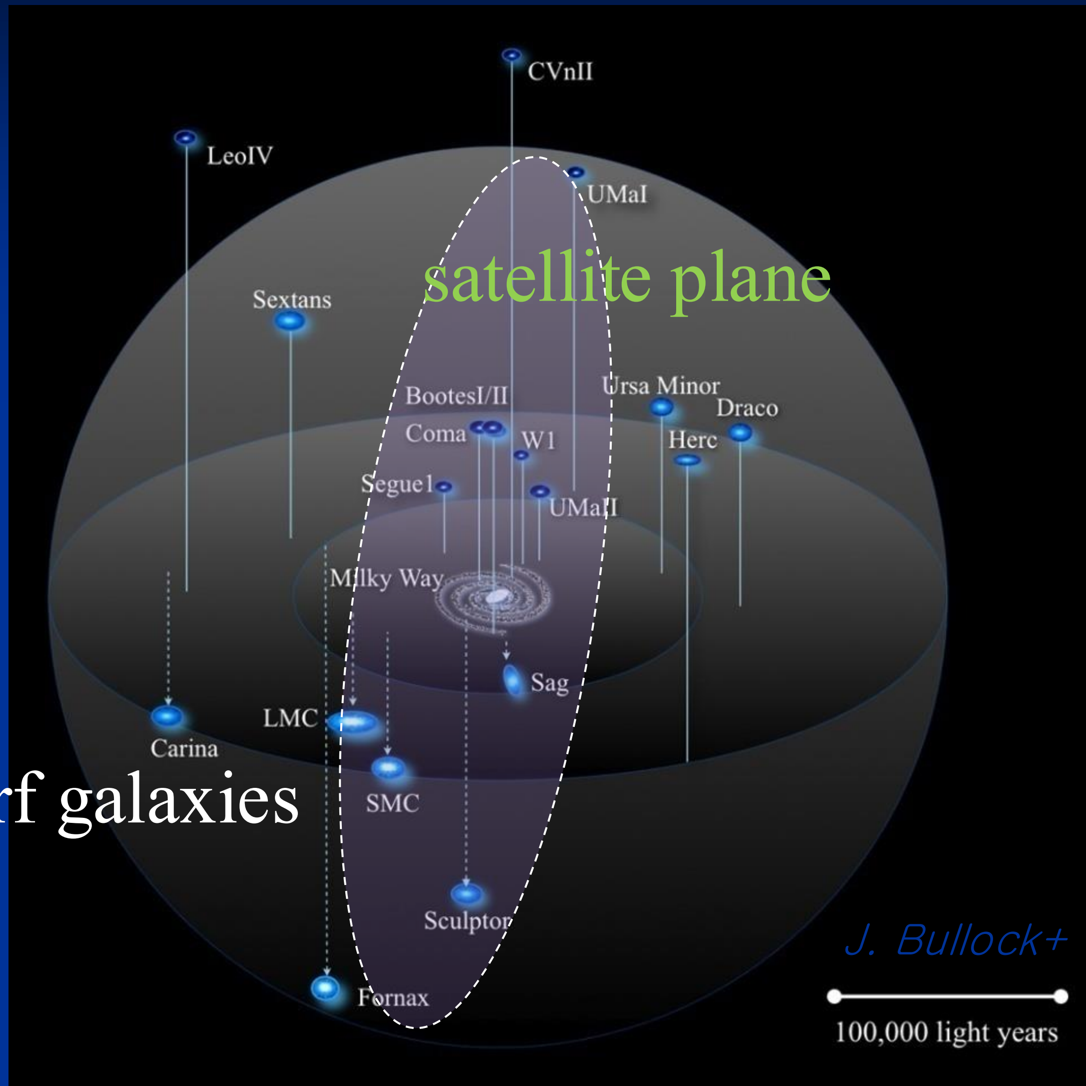


ULDM = Fuzzy, ULA, BEC, Wave, Scalar Field, ψ, Superfluid, quantum … ← PNGB, moduli, dilaton, ALP…

Compact objects in the mass range from $1.3 \times 10^{-5} M_\odot$ *to* 860 $M_\odot$ *cannot make up more than* 10% *of dark matter*. (Mroz+ 2403.02386)

→ No DM star or planet observed so far in galactic halos

Galaxies observed



Minimum mass
~ 10^6 Ms

Any good DM model should explain observed galaxies!

Small scale issues with CDM

Sales+ 2206.05295

ΛCDM Tensions with Dwarf Galaxies

No tension	Uncertain	Weak tension	Strong tension
			no DM model solved
√ Missing satellites	$M_\star$-$M_{\rm halo}$ relation	√ Too big to fail	Diversity of rotation curves
BTFR, L catastrophe	√ Core-cusp	Diversity of dwarf sizes	√ Satellite planes
	WDM	Quiescent fractions	Park+ Jcap 2022

- Key problem is how to suppress small scale structures < dwarf galaxies.
  → we need a new CDM → ULDM with m~ 10^{-22} eV might solve many of these
- Still unsolved problems seem to be related to Baryon-DM relation
- Can baryon physics + more precise numerical simulation + more observations save CDM?

Solutions to Small scale problems

- CDM : m ~ G eV (can't solve the problems)
  → need baryon physics (SN, BH jets,…)

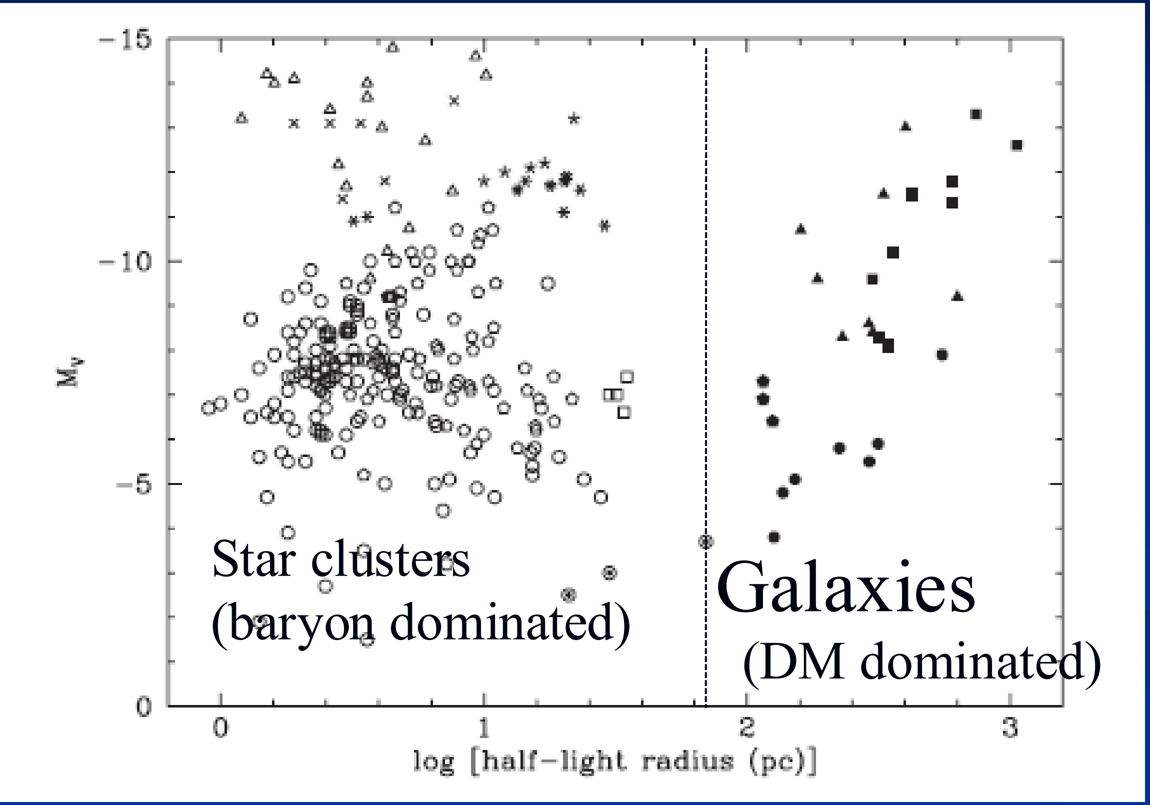
$* \ \sigma/\mathrm{m} \approx 10^{-26} \quad \mathrm{cm}^2/\mathrm{g}$ (100 GeV WIMP)

- SIDM: σ/m ~ 0.5-1 (Cluster), 5 (dwarf) [cm^2/g],
  → velocity dependent σ?

- WDM : m ~ k eV
  → Catch 22 problem (cusp)
  Suppression of the power spectrum prohibiting the formation of the dwarf galaxy

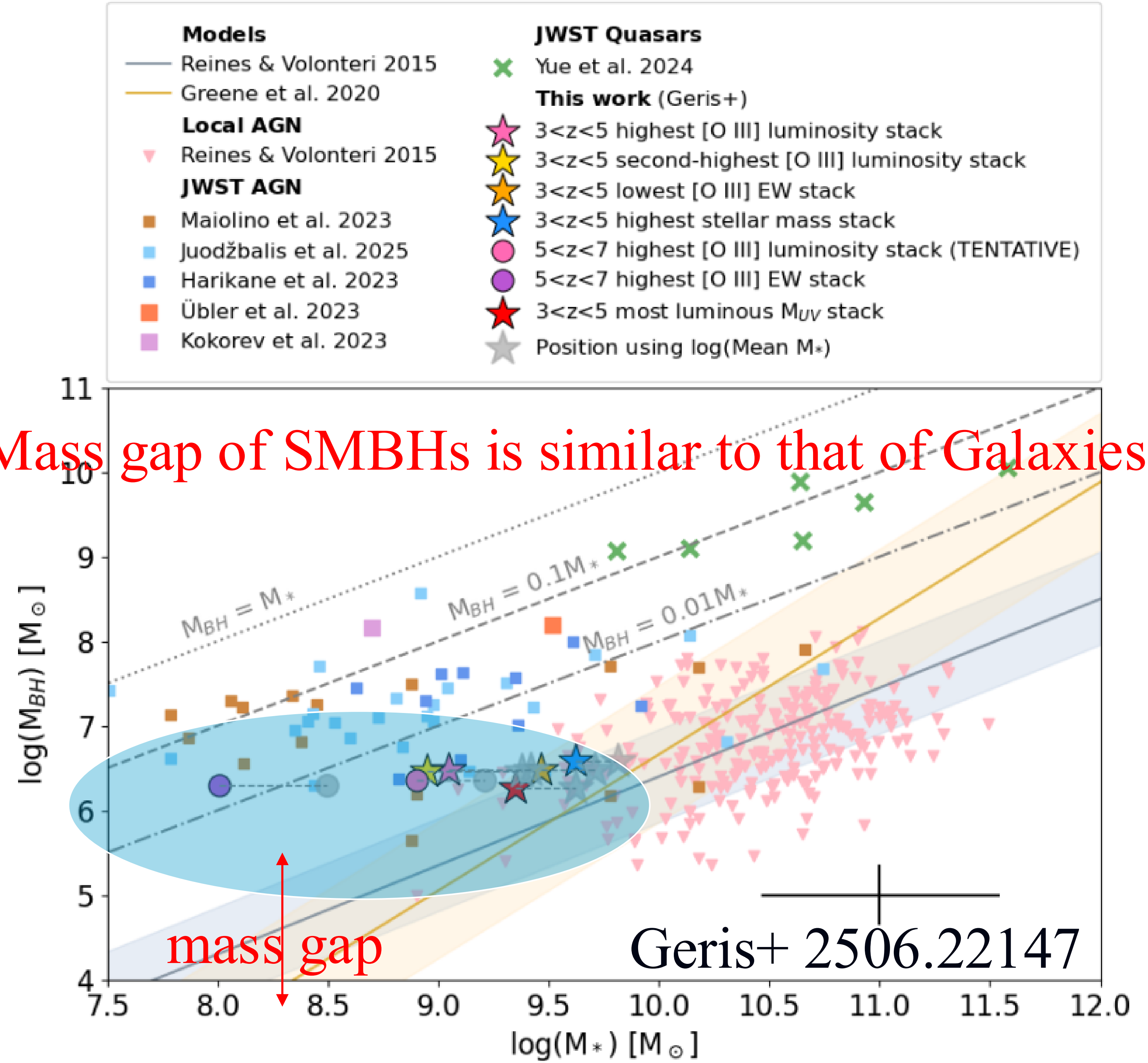
- ULDM: m ~ 10^{-22} eV
  → Lyman alpha favors m > 10^{-21} eV ?

Characteristic size & mass of dwarf galaxies



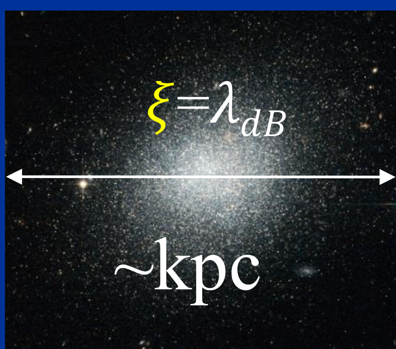
- There are no known stable galaxies with half-light radius smaller than 120 *pc* while the maximum size for star clusters deprived of DM is about 30 *pc*.
- The minimal observed mass of a stable dwarf galaxy is on the order of 10^7 Ms.
- The minimal observed mass of SMBHs is on the order of 10^5 Ms.
- Max mass of SMBHs and galaxies~ 10^{11} Ms.

→ *What is the origin of these scales (gaps)?*

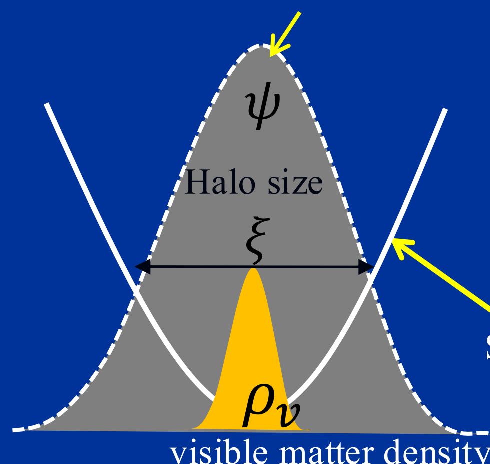

Models
Reines & Volonteri 2015
Greene et al. 2020
Local AGN
Reines & Volonteri 2015
JWST AGN
Maiolino et al. 2023
Juodžbalis et al. 2025
Harikane et al. 2023
Übler et al. 2023
Kokorev et al. 2023
JWST Quasars
Yue et al. 2024
This work (Geris+)
3<z<5 highest [O III] luminosity stack
3<z<5 second-highest [O III] luminosity stack
3<z<5 lowest [O III] EW stack
3<z<5 highest stellar mass stack
5<z<7 highest [O III] luminosity stack (TENTATIVE)
5<z<7 highest [O III] EW stack
3<z<5 most luminous M_{UV} stack
Position using log(Mean M_*)
Mass gap of SMBHs is similar to that of Galaxies
$M_{BH} = M_*$
$M_{BH} = 0.1M_*$
$M_{BH} = 0.01M_*$
mass gap
Geris+ 2506.22147
$\log(M_{BH})$ [$M_\odot$]
$\log(M_*)$ [$M_\odot$]

Galactic DM halos in ULDM

- Galactic DM halos (cores) are made of BEC ultra-light bosons described by macroscopic wave fn (similar to solitons).
- Quantum pressure (from uncertainty principle) prevents collapse
- Acts as CDM on super-galactic scales and solves small scale crisis of CDM



DM wave fn. of halo



uncertainty principle $\quad \xi \times p > \hbar$

min. halo size $\xi \approx \lambda_{dB} = \dfrac{h}{mv} \sim kpc$

$\rightarrow m \sim 10^{-22} eV \quad$ for $v \sim 10$km/s

Schroedinger -Poission Eq (SPE), nonlinear

$$\begin{cases} i\hbar\partial_t\psi = -\dfrac{\hbar^2}{2m}\nabla^2\psi + mV\psi \\ \nabla^2 V = 4\pi G(\rho_d + \rho_v), \end{cases}$$

DM density

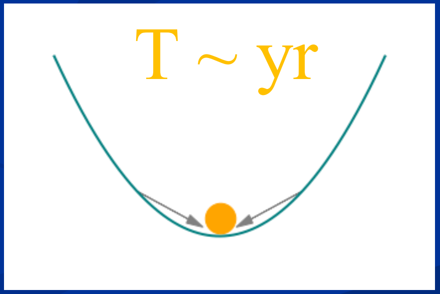
$$\rho_d = m^2|\psi|^2$$

Features of ULDM

$$\phi(t,x) = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi(t,x) + e^{imt}\psi^*(t,x)\right]$$

fast (bg), T~yr   slow (galaxy) T~ Myr

- Typical galaxy size ~ λ_{dB}~ kpc
  → min. mass & max mass of galaxies
- wave nature → gravitational cooling, interf.
- small dynamical friction
- bg oscillation with f~ m ~ nHz
- to explain DM relic density
  → GUT scale field value



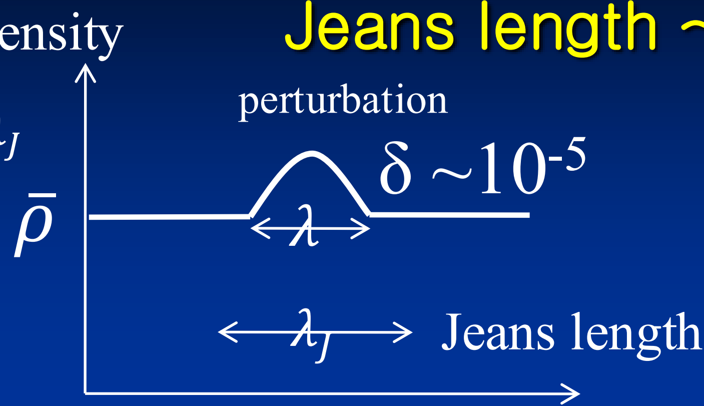
→ can explain many mysteries of galaxies

How did cosmic structures form?:

Jeans length ~ sound v. (c_s) * ff time

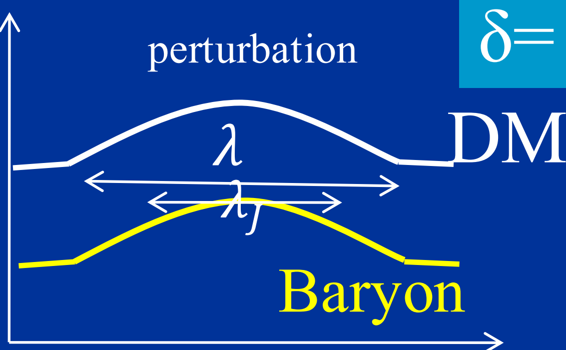
pressure

1) $\lambda < \lambda_J$

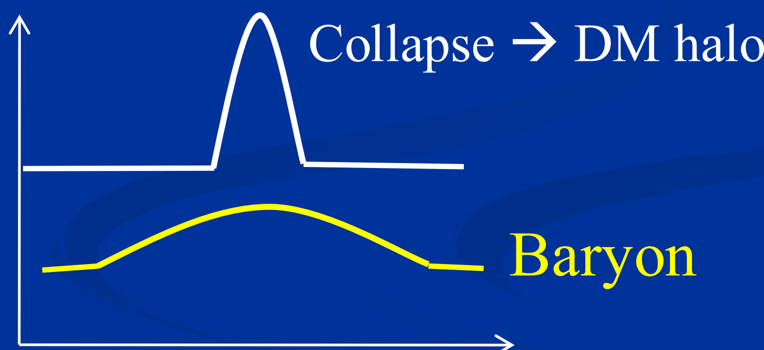


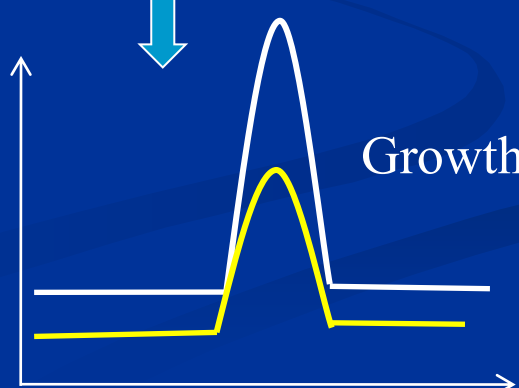
Oscillate & disperse

2) $\lambda > \lambda_J$



$\delta = \delta\rho / \bar{\rho}$





fluid

$$(\partial_t^2 - c_s^2 \nabla_r^2)\delta\rho = 4\pi G \bar{\rho} \delta\rho$$

$$k_J = \frac{\sqrt{4\pi G \bar{\rho}}}{c_s} = 2\pi / \lambda_J$$

Jeans length

Linear pert. Of ULDM

FDM has only 2 parameters m and bg density ρ_0 $(or\ F)$
(+ λ for ϕ^4 self-interacting ULDM)

a=scale factor

Nonrelativistic $\longrightarrow$

$$i\hbar\left(\frac{\partial\psi}{\partial t}+\frac{3}{2}H\psi\right)=-\frac{\hbar^2}{2ma^2}\Delta\psi+mV\psi+\frac{\lambda|\psi|^2\psi}{2m^2}$$ self-int

Madelung representation

$$perturbation \quad \text{with } \psi=\sqrt{\rho}e^{iS}, \qquad v\equiv\frac{\hbar}{ma}\nabla S\Rightarrow$$

$$\begin{cases}\partial_t\rho+3H\rho+\frac{1}{a}\nabla\cdot(\rho v)=0\\ \partial_t v+\frac{1}{a}v\cdot\nabla v+Hv+\frac{1}{\rho a}\nabla p+\frac{1}{a}\nabla V+\frac{\hbar^2}{2m^2a^3}\nabla\left(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\right)=0\end{cases}$$

Quantum Pressure

perturbation $\delta=\delta_k=\delta\rho/\rho_0$

Density contrast (k space)

$$\Rightarrow\partial_t^2\delta+2H\partial_t\delta+\left(\left(\frac{\hbar^2k^2}{4m^2a^2}+c_s^2\right)\frac{k^2}{a^2}-4\pi G\rho_0\right)\delta=0$$

Hubble drag

gravity

Quantum Jeans length

$$\lambda_{QJ}=\frac{2\pi}{k_J}a=\pi^{3/4}\hbar^{1/2}(G\rho_0m^2)^{-1/4}\propto 1/\sqrt{mH}$$ ~ 10 kpc

- CDM-like on super-galactic scale (for a small $k < k_J$)
- Suppress sub-galactic structure (for a large $k > k_J$)
  → ULDM is an ideal alternative to CDM

Galaxy mass from DM particle mass m

SIDM (particle) $M_J = \frac{\pi\bar{\rho}}{6}\left(\frac{\pi c_S^2}{G\bar{\rho}}\right)^{3/2}$ ← $\lambda_J = c_S\sqrt{\frac{\pi}{G\bar{\rho}}}$ ← $c_S = \sqrt{\frac{dp}{d\rho}}$

$observation \quad \sigma/m \sim {cm^2}/{g}$

Baryon (H) $M_J \equiv \frac{4\pi}{3}\rho_0\left(\frac{1}{2}\lambda_J\right)^3 = 26M_\odot\left(\frac{T}{10\ \mathrm{K}}\right)^{3/2}\left(\frac{n}{10^3\ \mathrm{cm}^{-3}}\right)^{-1/2}$

ULDM (fuzzy)

$M^Q{}_J = \frac{4\pi}{3}\rho(z)\lambda_{QJ}^3(z)$ Lee & Lim 0812.1342

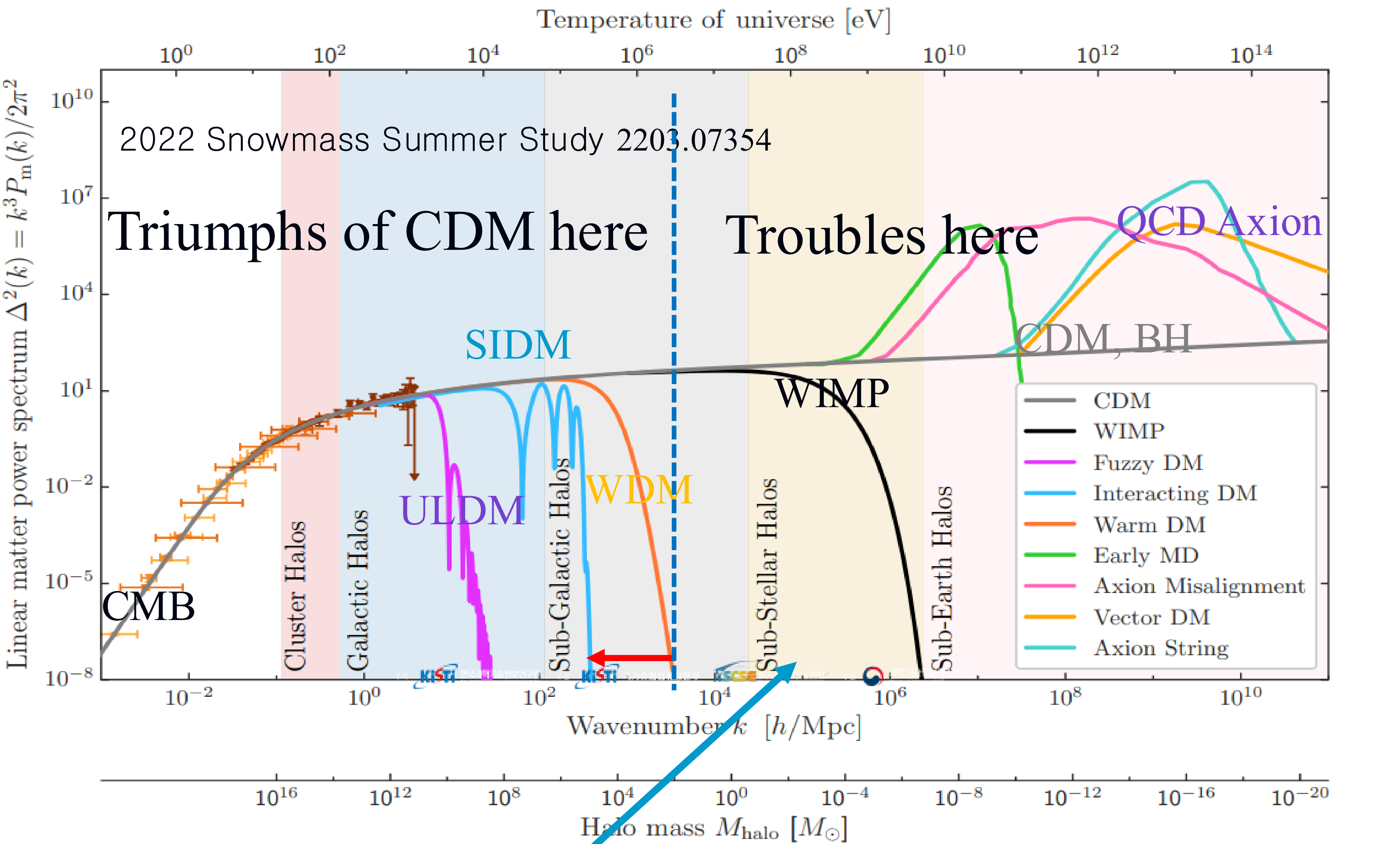
$= 1.204 \times 10^8\left(\frac{1+z}{m_{22}^2}\right)^{3/4}\left(\frac{\Omega_{\mathrm{dm}}}{0.27}\right)^{1/4}\left(\frac{h}{0.7}\right)^{1/2} M_\odot$

$m_{22} \equiv \frac{m}{10^{-22}\mathrm{eV}}$

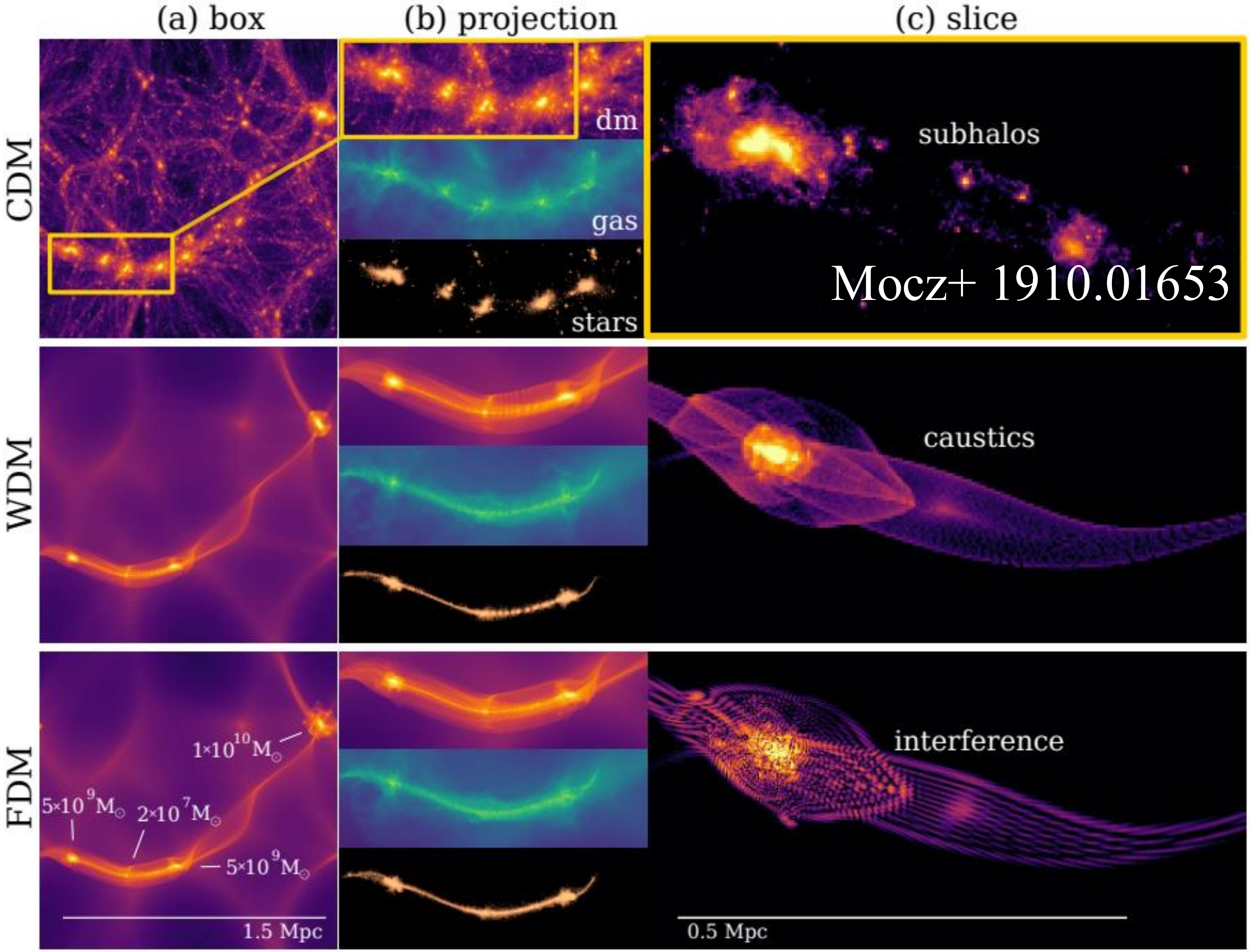
CDM, WDM collisionless

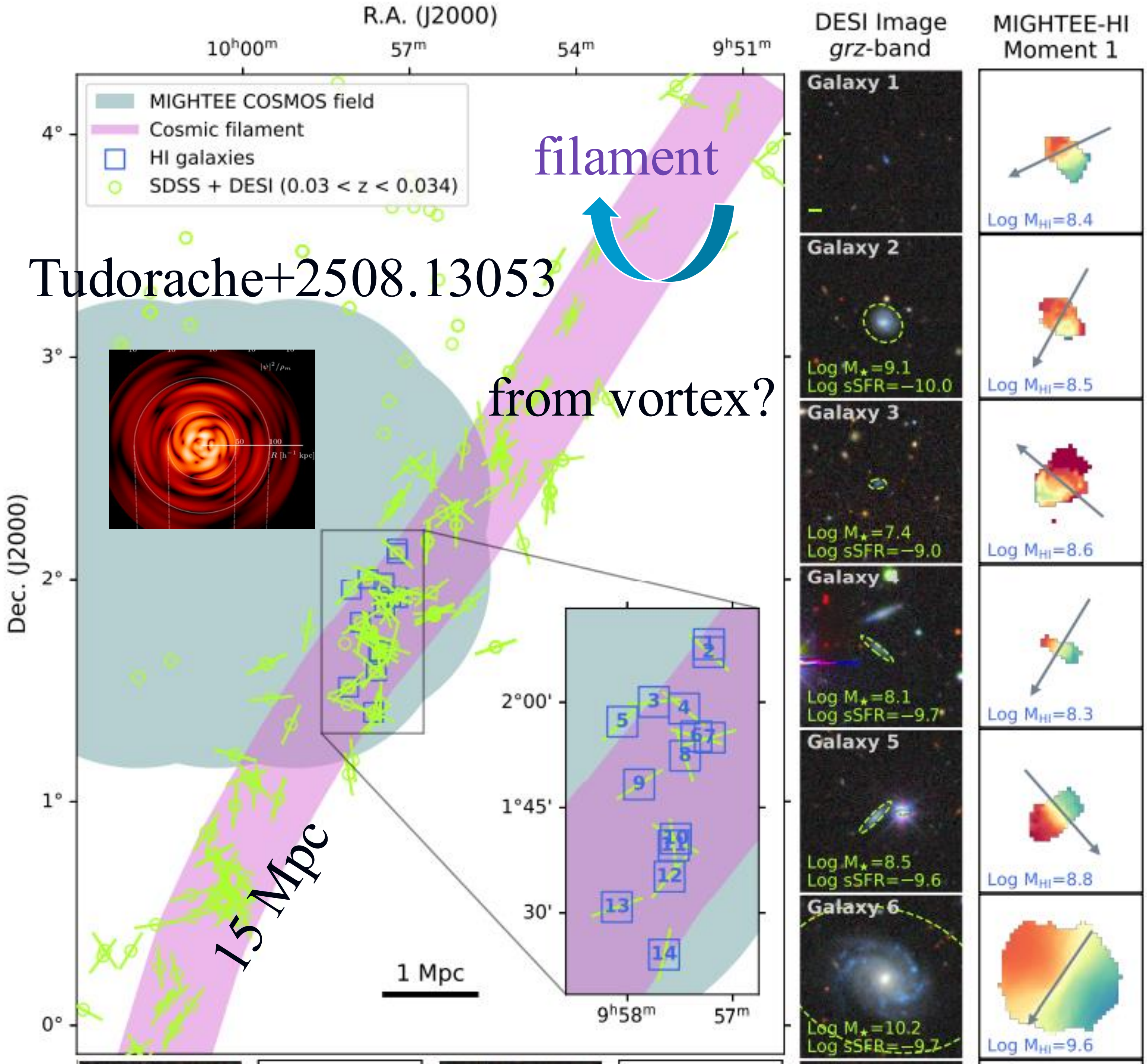
$M_{fs} = 10^{10}\left(\frac{m}{1keV}\right)^{-3.33} Ms$

Galaxies are DM dominated and seem to have kpc size scale



No cuspy subhalo, DM star or DM planet found so far
→ DM has kpc length scale?

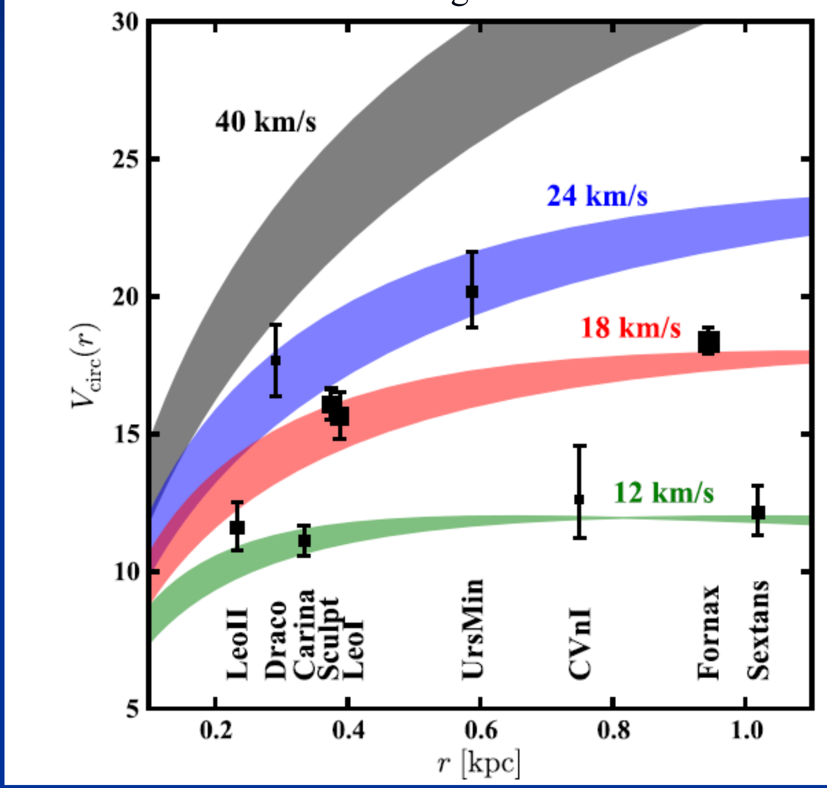

(a) box
(b) projection
(c) slice
CDM
WDM
FDM
dm
gas
stars
subhalos
caustics
interference
Mocz+ 1910.01653
1×10^10M⊙
5×10^9M⊙
2×10^7M⊙
5×10^9M⊙
1.5 Mpc
0.5 Mpc

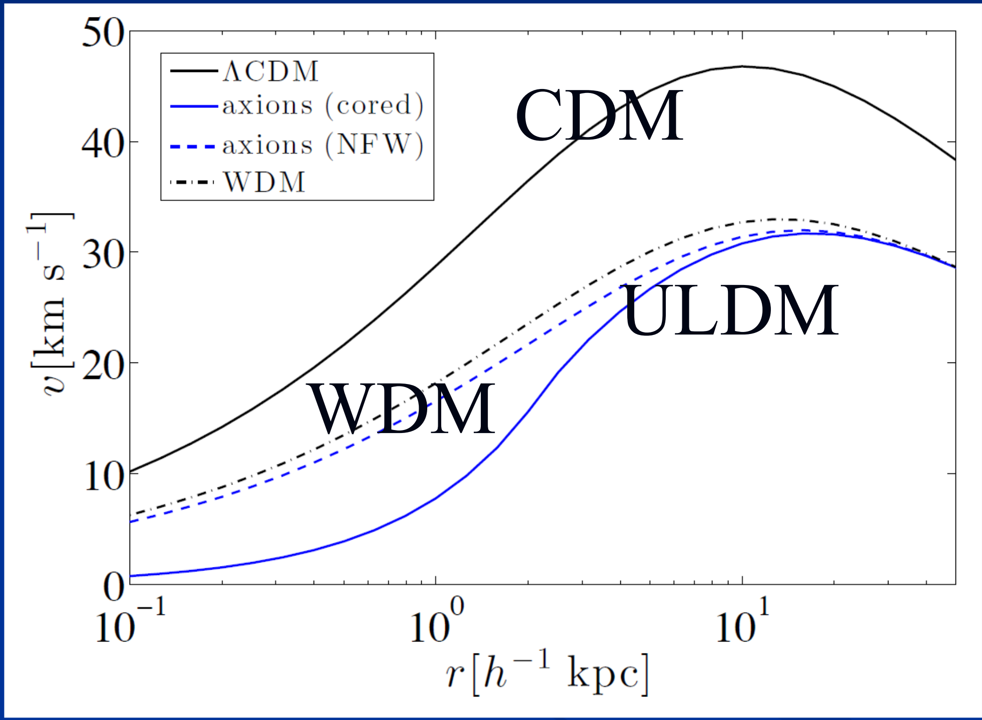


Tudorache+2508.13053
filament
from vortex?
15 Mpc
R.A. (J2000)
10h00m
57m
54m
9h51m
Dec. (J2000)
4°
3°
2°
1°
0°
MIGHTEE COSMOS field
Cosmic filament
HI galaxies
SDSS + DESI (0.03 < z < 0.034)
1 Mpc
2°00'
1°45'
30'
9h58m
57m
1
2
3
4
5
6
7
8
9
10
11
12
13
14
DESI Image
grz-band
MIGHTEE-HI
Moment 1
Galaxy 1
Log M_HI=8.4
Galaxy 2
Log M★=9.1
Log sSFR=−10.0
Log M_HI=8.5
Galaxy 3
Log M★=7.4
Log sSFR=−9.0
Log M_HI=8.6
Galaxy 4
Log M★=8.1
Log sSFR=−9.7
Log M_HI=8.3
Galaxy 5
Log M★=8.5
Log sSFR=−9.6
Log M_HI=8.8
Galaxy 6
Log M★=10.2
Log sSFR=−9.7
Log M_HI=9.6

Too big to fail= no dense satellite

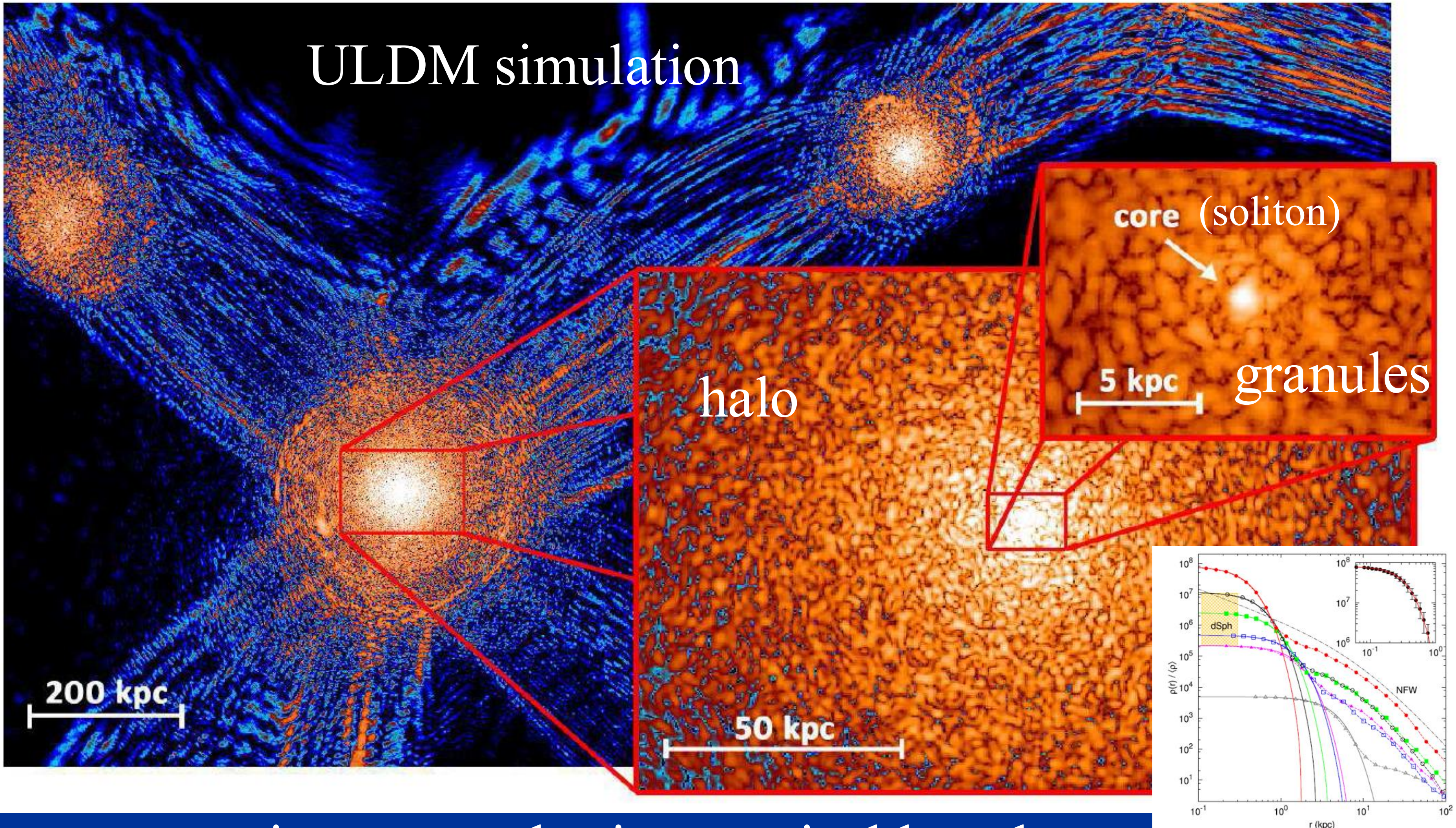
Where are these bright satellites?





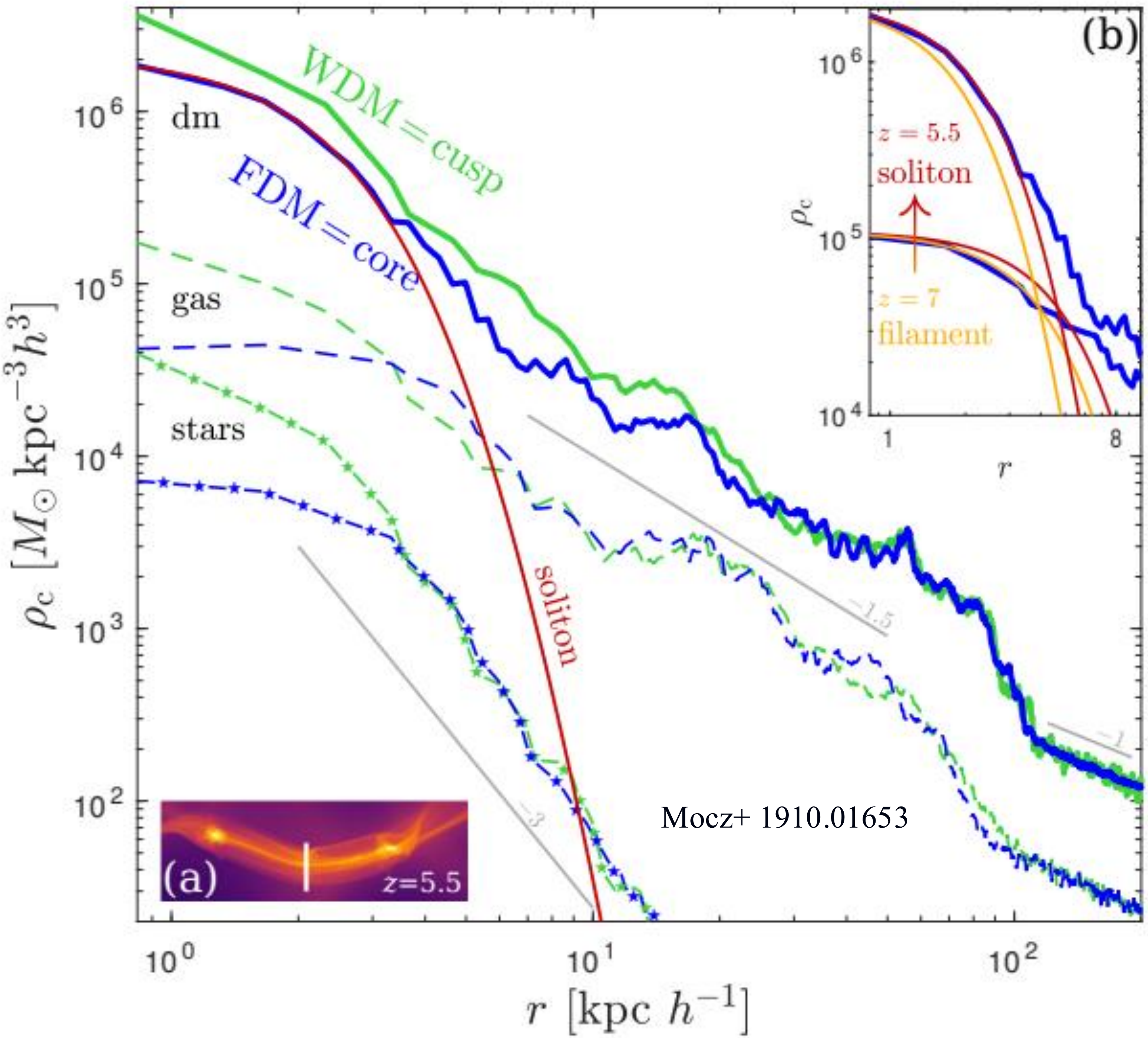
Marsh & Silk 2013

Boylan etal 2012



core size ~ granule size~ typical length
~ Q. Jeans length (kpc)

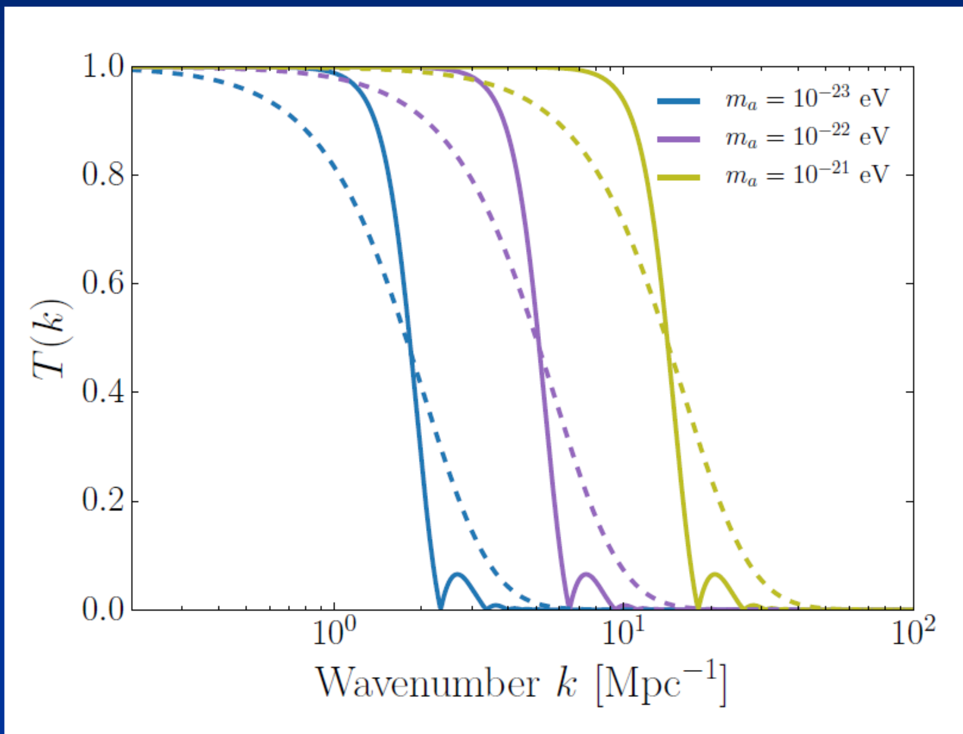
Schive+ , Nature physics 2014



$\rho_c\ [M_\odot\ \mathrm{kpc}^{-3} h^3]$
$r\ [\mathrm{kpc}\ h^{-1}]$
dm
gas
stars
WDM = cusp
FDM = core
soliton
−1.5
−3
−1
(a)
z=5.5
(b)
$z = 5.5$
soliton
$z = 7$
filament
ρ_c
r
Mocz+ 1910.01653

Can FDM solve small scale issues?

Hu et al PRL 2000 (Fuzzy DM)



$P_{FDM} = T_F^2(k) P_{CDM}(k)$

$T_F^2(k) \approx \frac{\cos x^3}{1 + x^8}$

$x = 1.61 m_{22}^{1/18} \; k/k_J,$

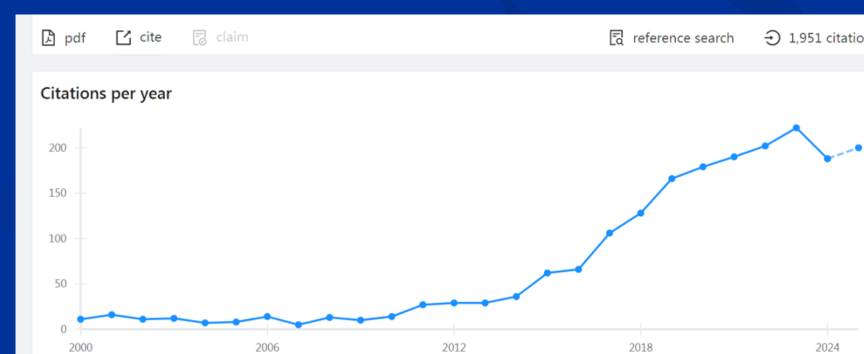
$m = m_{22} \times 10^{-22} eV$

The power drops by a factor of 2 at $4.5 m_{22}^{4/9} Mpc^{-1}$

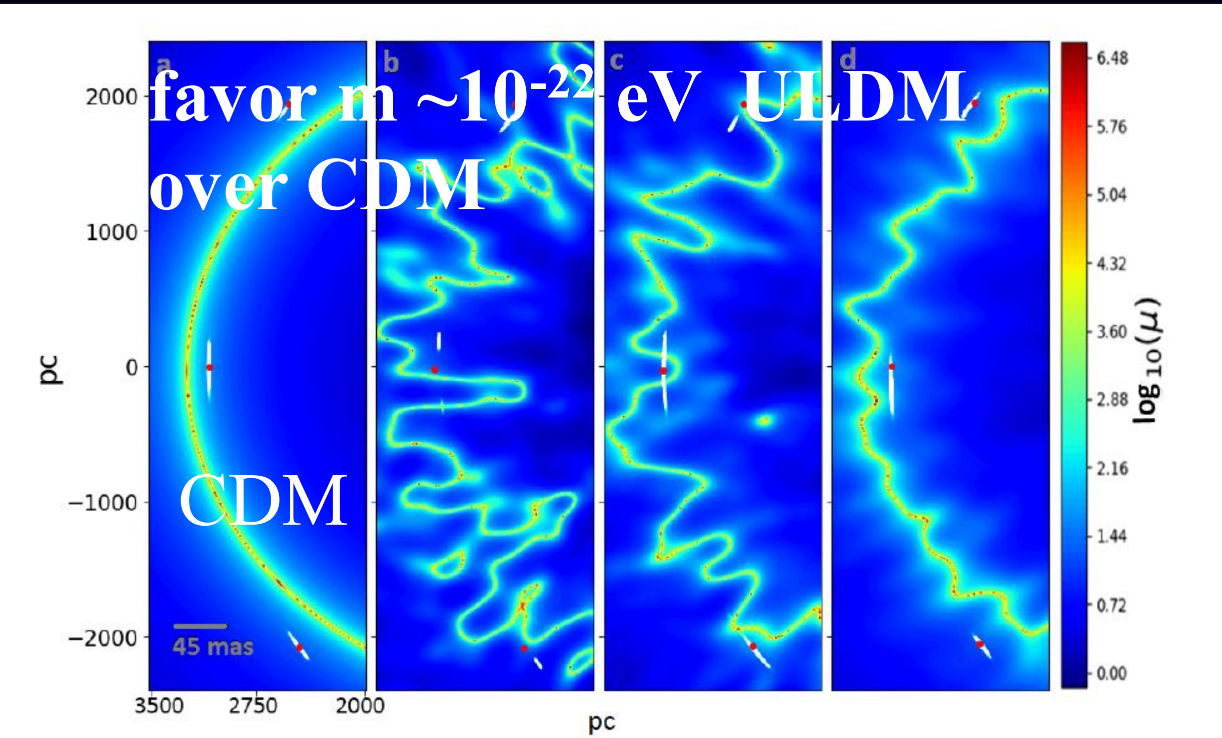
cutoff at k$\sim 4.5 hMpc^{-1}$

$\Rightarrow m \approx 10^{-22} eV!$

Marsh



To solve the cusp and missing satellites suppression at small scale needed

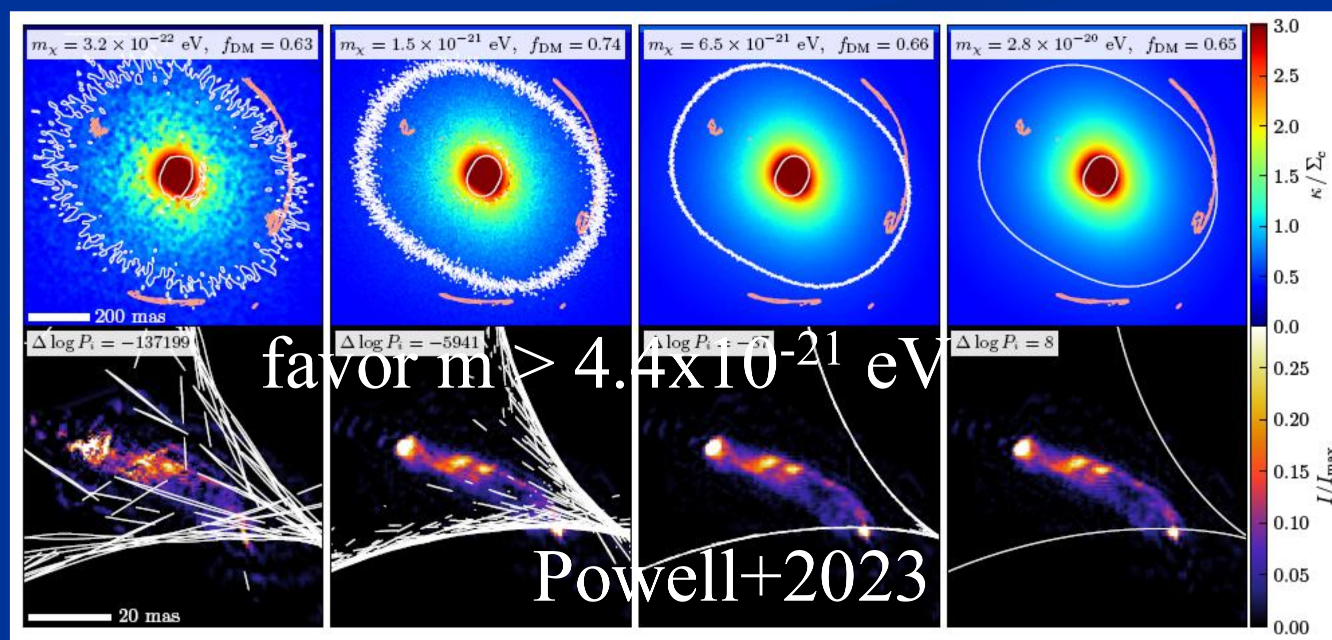


favor m ~10^{-22} eV ULDM over CDM

CDM

ULDM well reproduce lens of radio objects

Armurth+2023



favor m > 4.4x10^{-21} eV

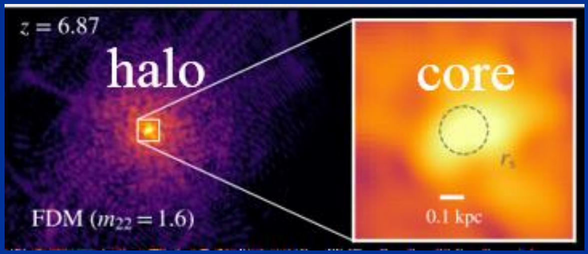
Powell+2023

core–halo relation of ULDM

$$Q.\,Jeans\ mass$$

$$M^{Q}{}_{J} = \frac{4\pi}{3}\rho(z)\lambda_{QJ}^{3}(z) = 1.204\times 10^{8}\left(\frac{1+z}{m_{22}^{2}}\right)^{3/4}\left(\frac{\Omega_{\rm dm}}{0.27}\right)^{1/4}\left(\frac{h}{0.7}\right)^{1/2} M_{\odot}$$

$\sim M_h \quad halo\ mass$



DM core size

$$r_s = 0.135 m_{22}^{-1}\left(\frac{\zeta'(z)}{\zeta'(7)}\right)^{-1}\left(\frac{M_h}{10^{11}M_{\odot}}\right)^{-1/3}\ \text{kpc}$$

DM core mass

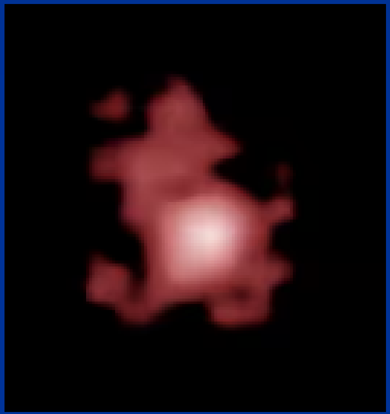
$$M_s = 1.68\times 10^{9} m_{22}^{-1}\left(\frac{\zeta'(z)}{\zeta'(7)}\right)\left(\frac{M_h}{10^{11}M_{\odot}}\right)^{1/3} M_{\odot} \sim \text{Mass gap}$$

where $m_{22} \equiv \frac{m}{10^{-22}\text{eV}}$, $\zeta' = (1+z)^{1/2}\zeta(z)^{1/6}$

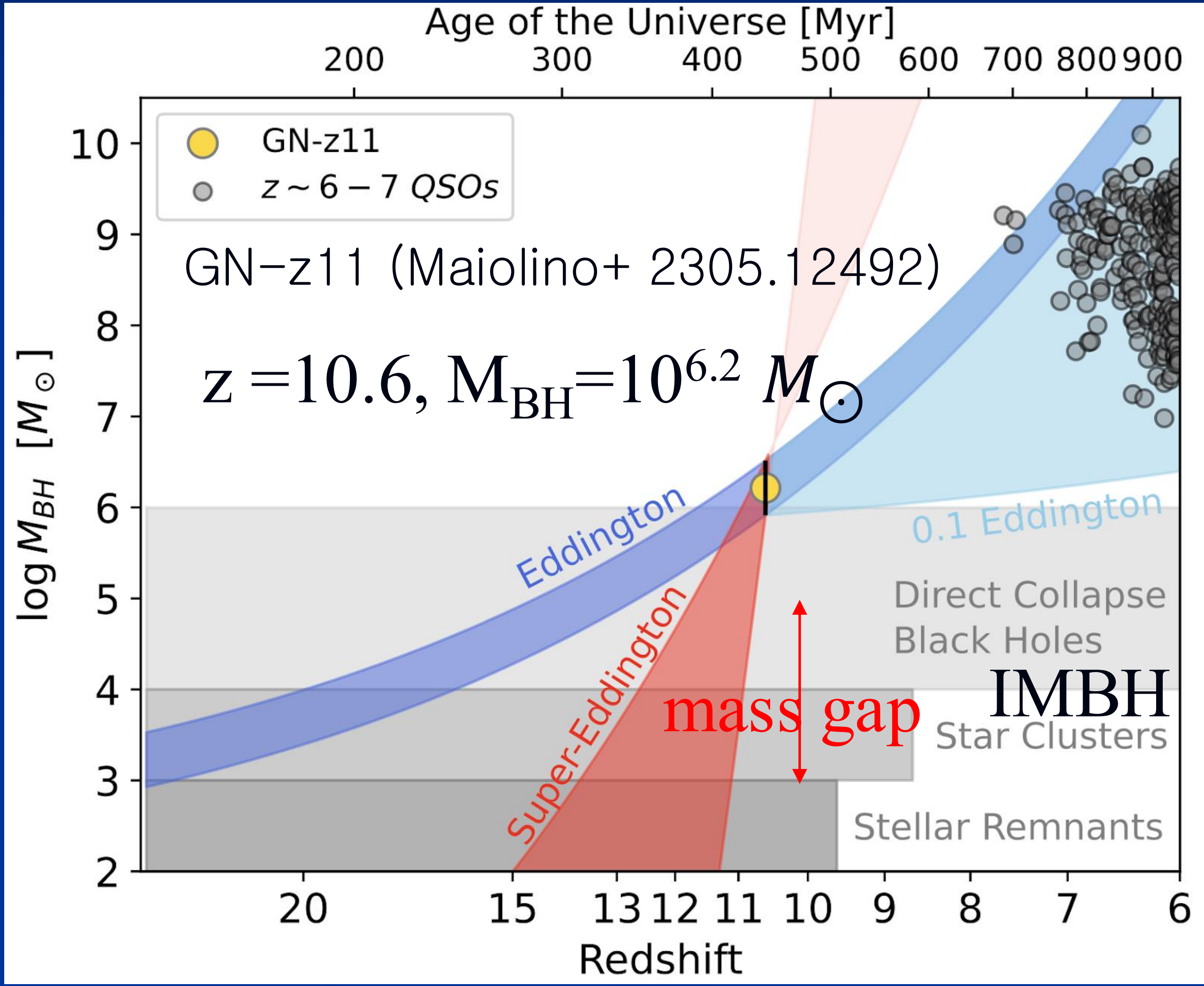
with $\zeta(z) = \left(18\pi^2 + 82(\Omega_m(z)-1) - 39(\Omega_m(z)-1)^2\right)/\Omega_m(z) \sim 180$ at $z \geq 1$

Impossibly early SMBH?

SMBHs (Quasar) with M~10^6 M_s were already mature at around z=11 (~0.45 Gyr)

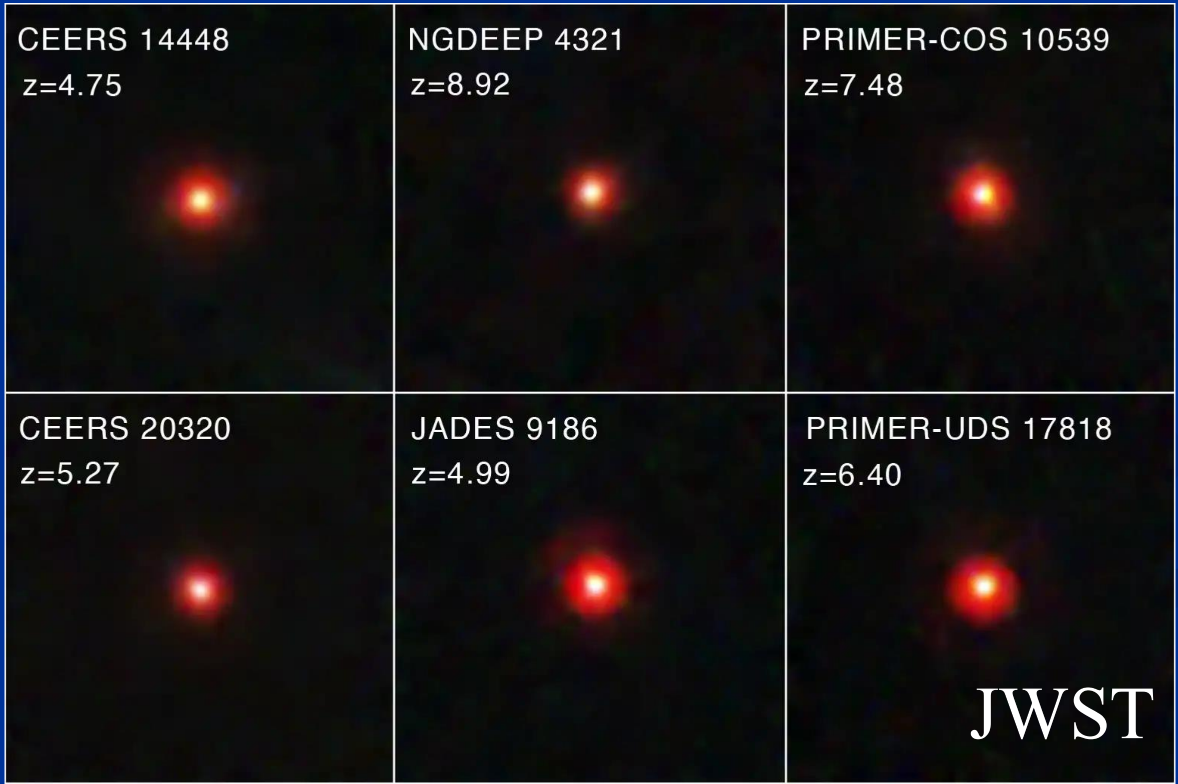


Cosmic chicken-egg problem



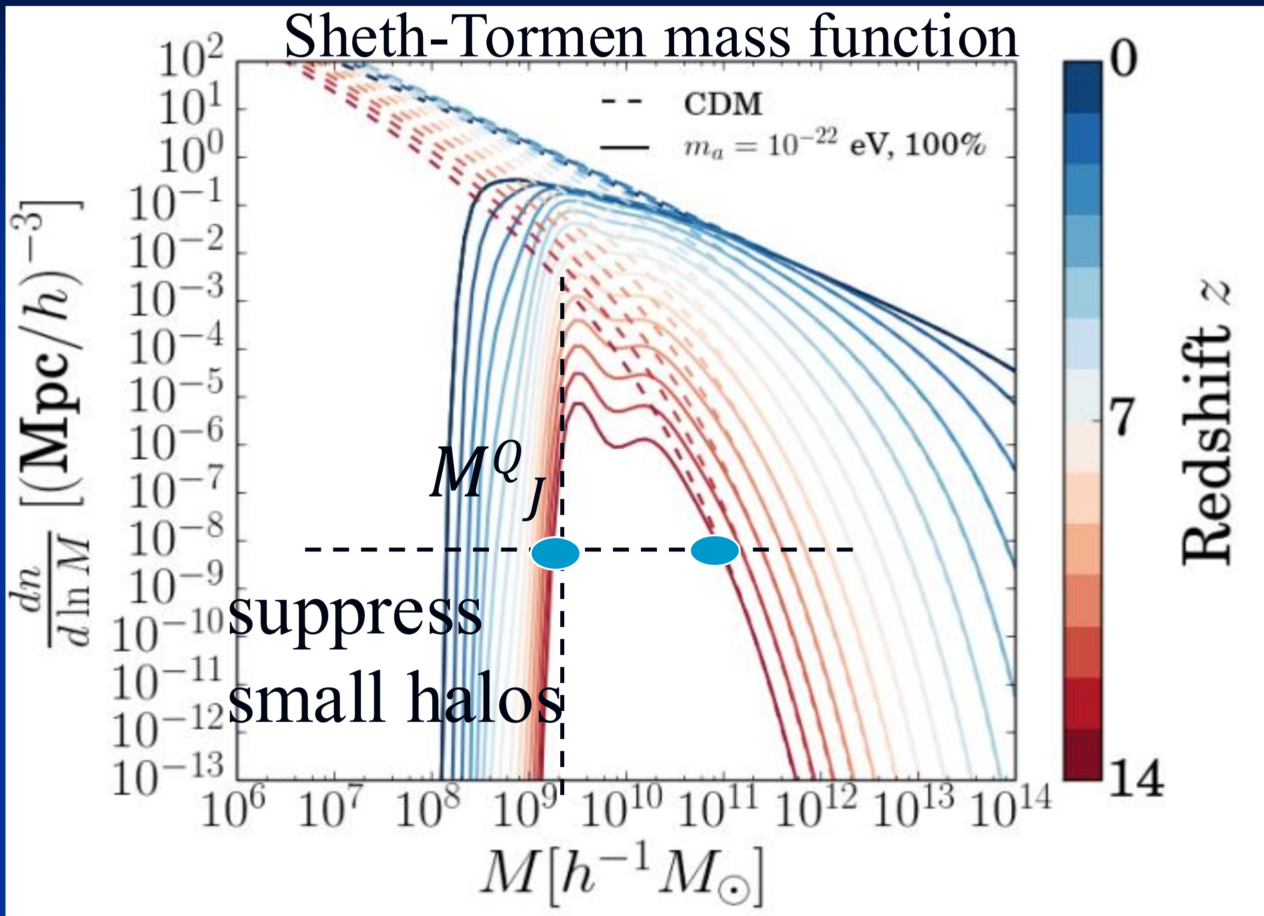
Little Red Dots

- Size: less than a few hundred light-years (smaller, older than quasar hosts)
- Color: Red; influenced by redshift and dust absorption?
- Epoch of Formation: Approximately 0.6–1.5 billion years after the Big Bang
- Central Structure: Rapid gas rotation → high likelihood of hosting an AGN
- Observational Features: Weak X-ray emission, no photometric variability
- Possible Models: Obscured AGN?, supermassive stars? Black hole stars?



Halo mass function

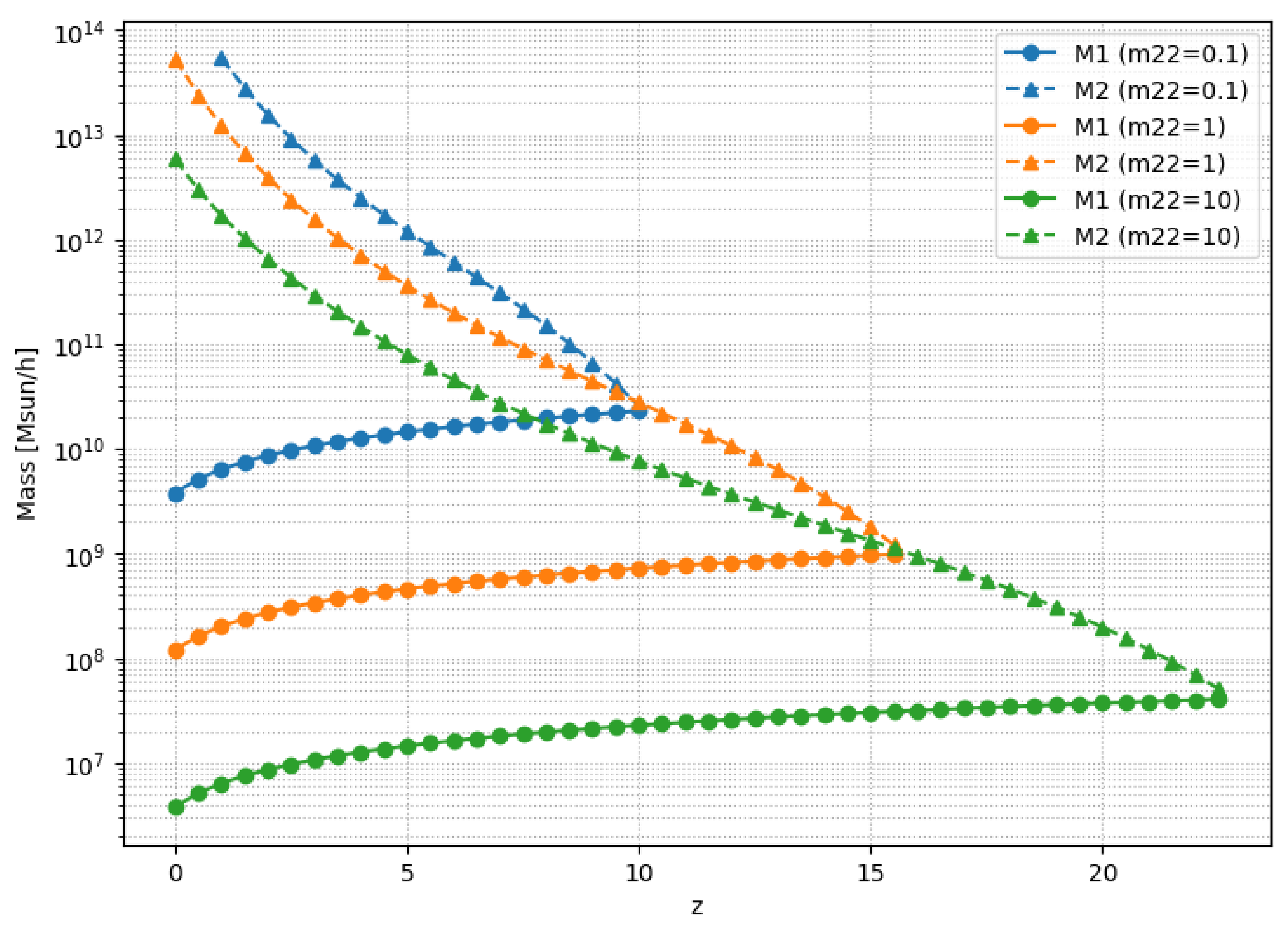
Bozek+ 1409.3544



$$M_h = \alpha^3 M^Q{}_J$$

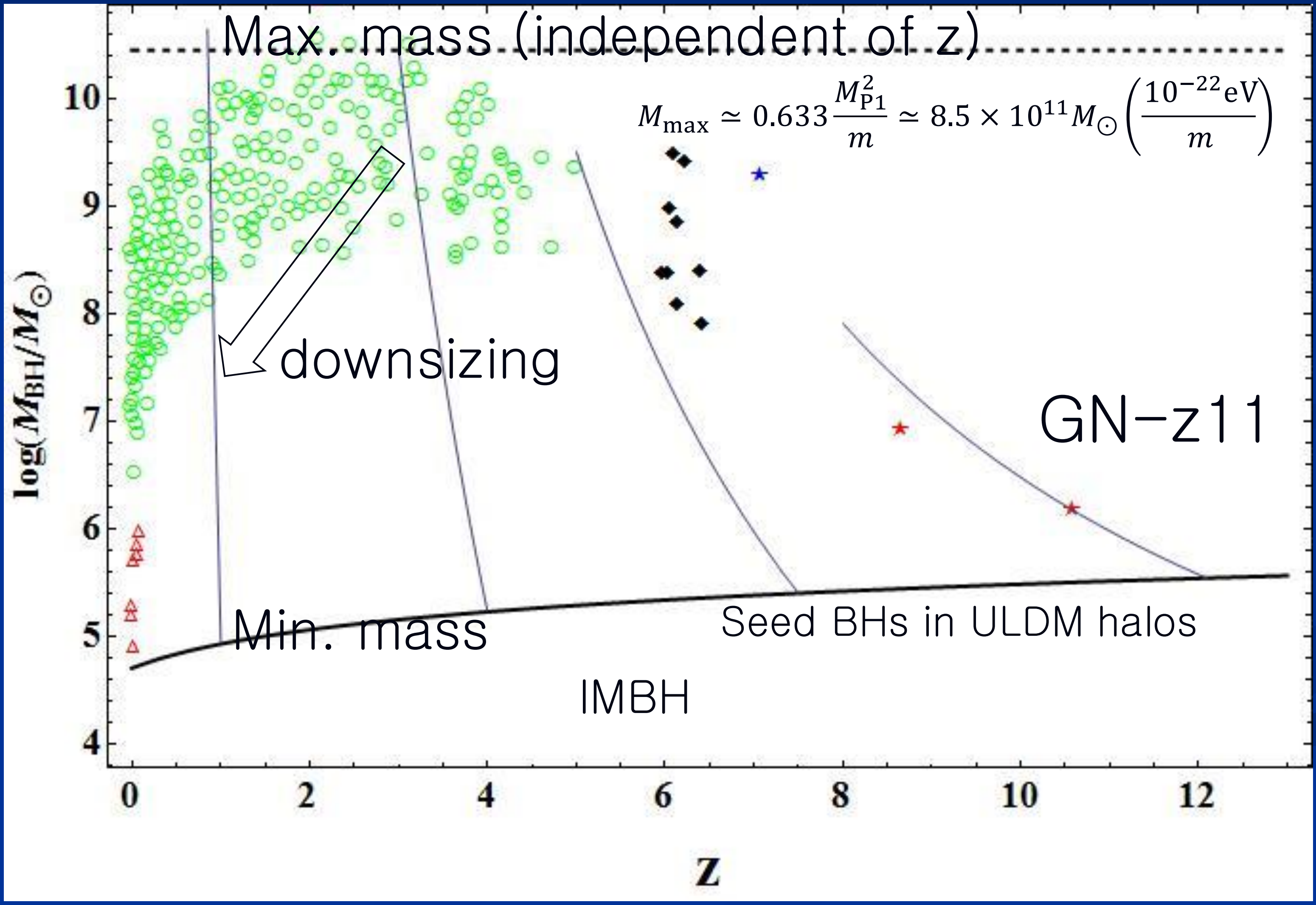
$$\left.\frac{dn}{d\ln M}\right|_{\mathrm{FDM}} = \left.\frac{dn}{d\ln M}\right|_{\mathrm{CDM}} \left[1 + \left(\frac{M_h}{M_0}\right)^{-1.1}\right]^{-2.2} \quad M_0 = 1.6 \times 10^{10} m_{22}^{-4/3} M_\odot$$

$$\left.\frac{dn}{d\ln M}\right|_{\mathrm{CDM}} \propto f(\nu) = A\sqrt{\frac{1}{2\pi}}\sqrt{q}\nu\left[1 + (\sqrt{q}\nu)^{-2p}\right]\exp\left(-\frac{q\nu^2}{2}\right)\frac{1}{\nu} \qquad \nu = \frac{1.69}{\sigma(M)}$$



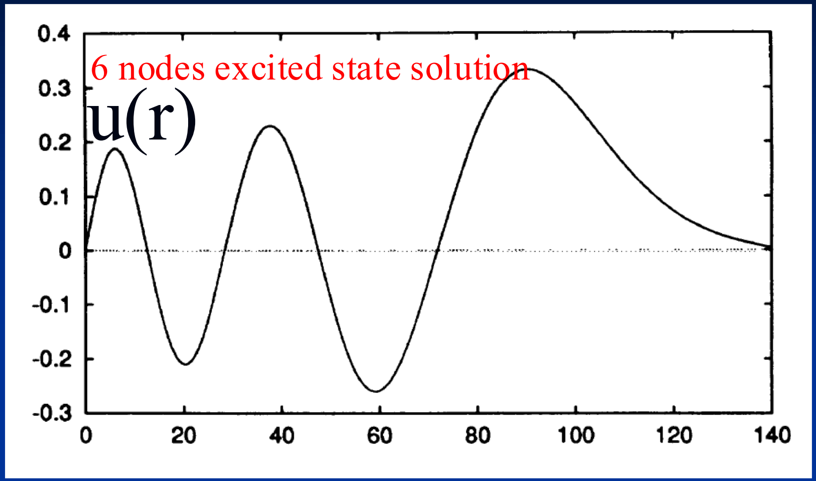
M1 (m22=0.1)
M2 (m22=0.1)
M1 (m22=1)
M2 (m22=1)
M1 (m22=10)
M2 (m22=10)
Mass [Msun/h]
10^{7}
10^{8}
10^{9}
10^{10}
10^{11}
10^{12}
10^{13}
10^{14}
0
5
10
15
20
z

Evolution of SMBHs from ULDM



BEC DM

S. Sin PRD 1994



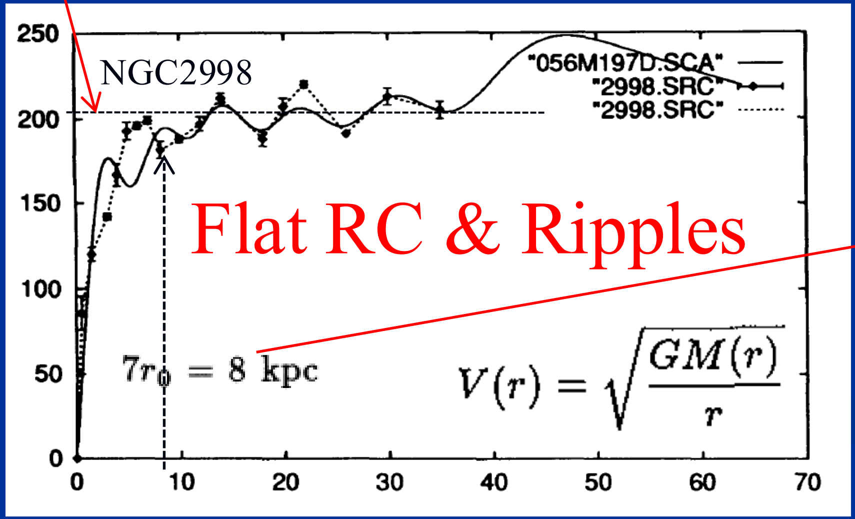
$$\hat{\psi}(\hat{r}) = \frac{1}{\sqrt{4\pi}} \frac{u(\hat{r})}{\hat{r}}$$ rescaled

$$v_{\mathbf{rotation}} \lambda_{\mathbf{ripple}} = \frac{1}{\sqrt{2}} \frac{1}{m} f(\hat{r})$$

$$f(\hat{r}) = \left(\int_0^{\hat{r}} d\hat{r} u^2(\hat{r})/\hat{r}[\hat{V}(\hat{r}_t) - \hat{V}(\hat{r})] \right)^{\frac{1}{2}}$$

$0.32 v_0 = 204 \text{ km/sec}$

$\rho_{\mathbf{dark}} = G M_0 m |\psi|^2$



Gravitational Bohr radius

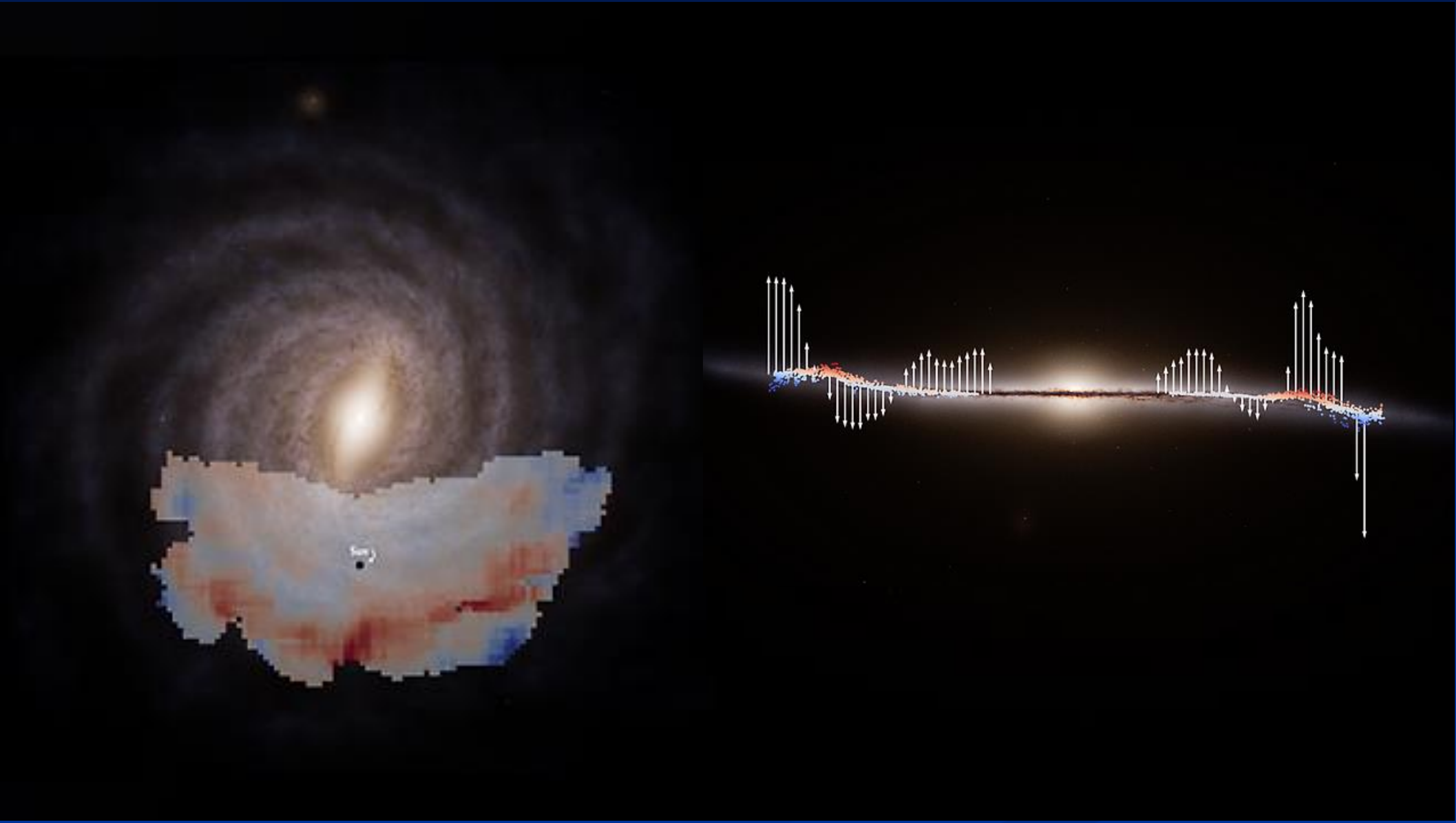
$r_0 = \hbar^2 / 2 G M_0 m^2$ $\Rightarrow$ $m = 3.3 \times 10^{-23} \text{eV}$

$M_0 = 0.69 \times 10^{12} M_\odot$

$v_0 = \sqrt{G M_0 / r_0}.$

Total mass $M = \hat{M} M_0 = 5.9 \times 10^{12} M_\odot$

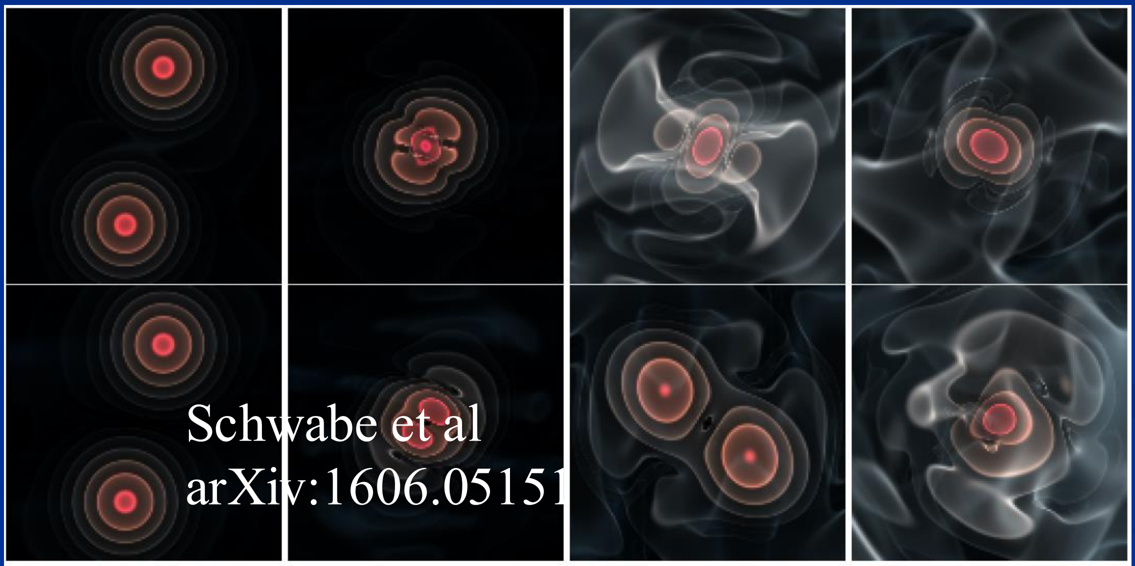
Warp, Ripple, Wave



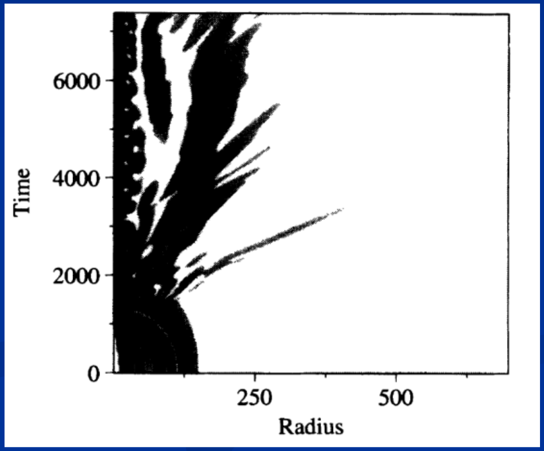
Poggio+ 2407.18659

Gravitational cooling of FDM

Unique and efficient dissipation mechanism of FDM



Schwabe et al
arXiv:1606.05151



Seidel, Suen PRL 72

DM halo (wave) collision → nonlinear interference
→ creation of high momentum (k) modes
→ escape of waves carrying E, p, L a way

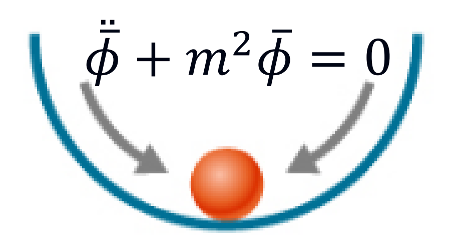
Oscillation of ULDM

$$\phi(\mathbf{x}, l) = \bar{\phi}(l) + \delta\phi(\mathbf{x}, l)$$

$$\omega = \frac{1}{2.5 months} \frac{m}{10^{-22} eV}$$

where

- $\bar{\phi}(l)$: spatially homogeneous cosmic background field,
- $\delta\phi(\mathbf{x}, l)$: inhomogeneous part forming nonlinear galactic halos.



For a scalar potential

$$V(\phi) = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{4}\phi^4$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2}\nabla^2\phi + m^2\phi + \lambda\phi^3 = 0$$

Inserting $\phi = \bar{\phi} + \delta\phi$ and spatially averaging, the background satisfies

$$\ddot{\bar{\phi}} + 3H\dot{\bar{\phi}} + m^2\bar{\phi} + \lambda\bar{\phi}^3 + \lambda\bar{\phi}\langle\delta\phi^2\rangle + \cdots = 0.$$

The local perturbation $\phi_h \equiv \delta\phi$ obeys

$$\ddot{\phi}_h + 3H\dot{\phi}_h - \frac{1}{a^2}\nabla^2\phi_h + m^2\phi_h + 3\lambda\bar{\phi}^2\phi_h + \cdots = 0$$

this simplifies to

$$\ddot{\phi}_h - \nabla^2\phi_h + m^2\phi_h + \lambda\phi_h^3 = 0.$$

ULA miracle

Hui et al 2017

$$I = \int d^4x\sqrt{g}\left[\frac{1}{2}F^2 g^{\mu\nu}\partial_\mu a\partial_\nu a - \mu^4(1-\cos a)\right]$$

$m = \frac{\mu^2}{F}$, $F =$ *typical field value*

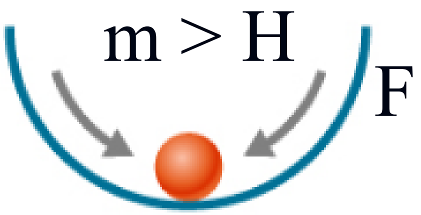
$\ddot{a} + 3H\dot{a} + m^2 \sin a = 0$

oscillation starts at $\mathrm{H} \sim \frac{T_{osc}^2}{M_P} = m$

MDE starts at $T_1 \sim 1eV \rightarrow \frac{\mu^4(DM)}{T_{osc}^4(rad)} \rightarrow \frac{\mu^4 T_{osc}}{T_{osc}^4 T_1} \sim 1$

$F = \frac{\mu^2}{m} \sim \frac{M_P^{3/4} T_1^{1/2}}{m^{1/4}} \sim 10^{17}GeV$ $\quad$ *typical field value*

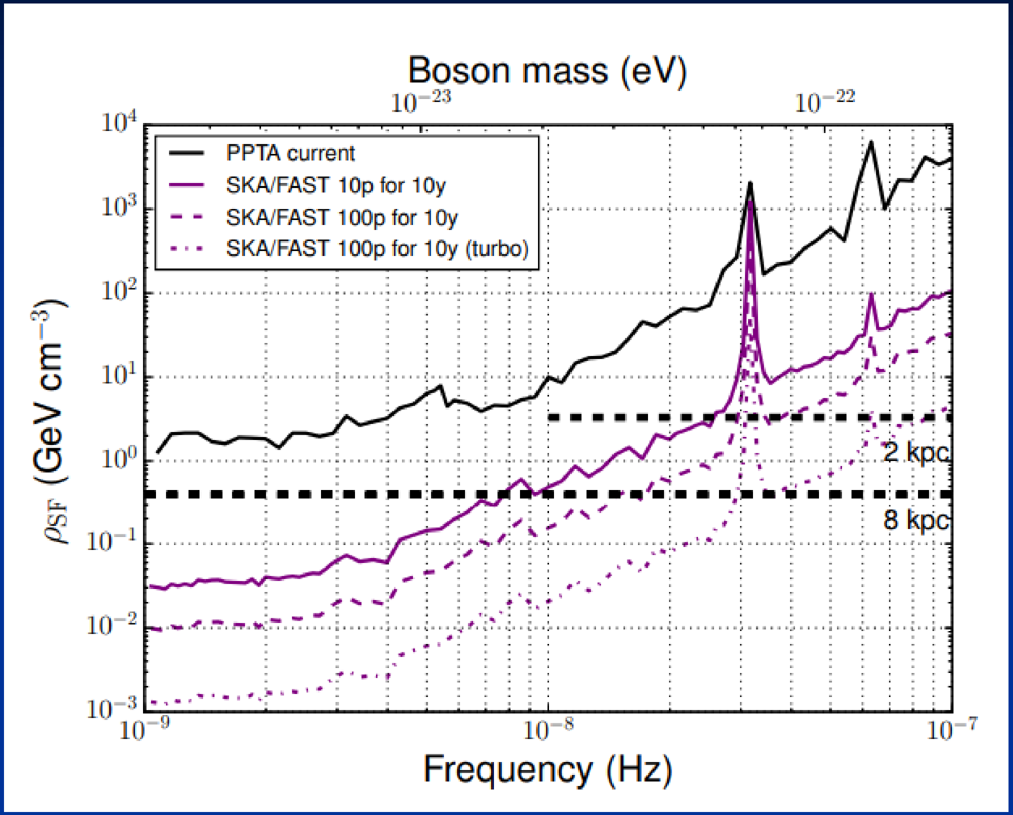
$\Omega_a \sim 0.1\left(\frac{F}{10^{17}GeV}\right)^2\left(\frac{m}{10^{-22}eV}\right)^{1/2}$ $\quad$ ULA miracle?



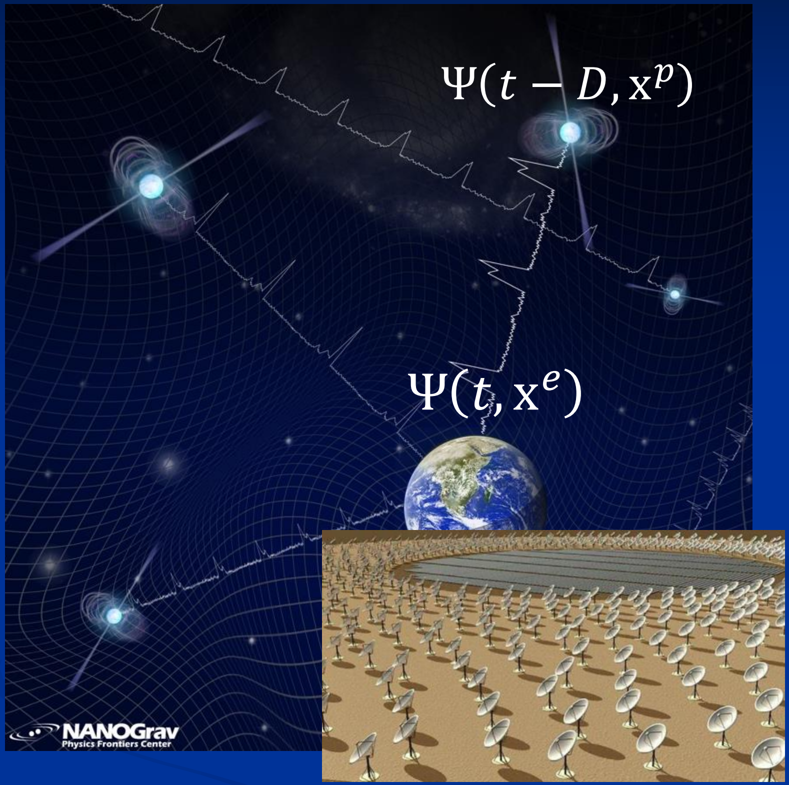
ULDM naturally explains DM density with GUT scale.

This holds for ULDM with a quadratic pot.

GW background detected by pulsar timing array

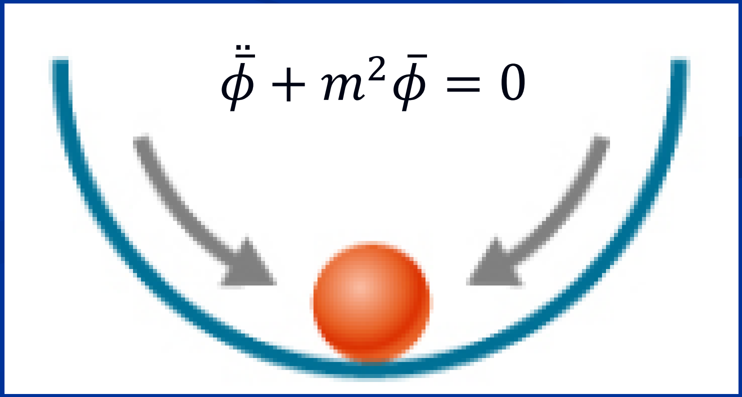


1810.03227



ULDM has
intrinsic osc time scale
1/m ~ yrs

$$\omega = \frac{1}{2.5months}\frac{m}{10^{-22}eV}$$



Gravitational atom

$$\left(g^{\alpha\beta}\nabla_\alpha\nabla_\beta - \mu^2\right)\Phi(t,r) = 0$$

ansatz

$$\Phi(t,r) = \frac{1}{\sqrt{2\mu}}\left[\psi(t,r)\mathrm{e}^{-\mathrm{i}\mu t} + \psi^*(t,r)\mathrm{e}^{\mathrm{i}\mu t}\right]$$

Schroedinger equation with a Coulomb-like central potential

$$\mathrm{i}\partial_t\psi(t,r) = \left(-\frac{1}{2\mu}\nabla^2 - \frac{\alpha}{r} + O(\alpha^2)\right)\psi(t,r)$$

hydrogen atom like solution

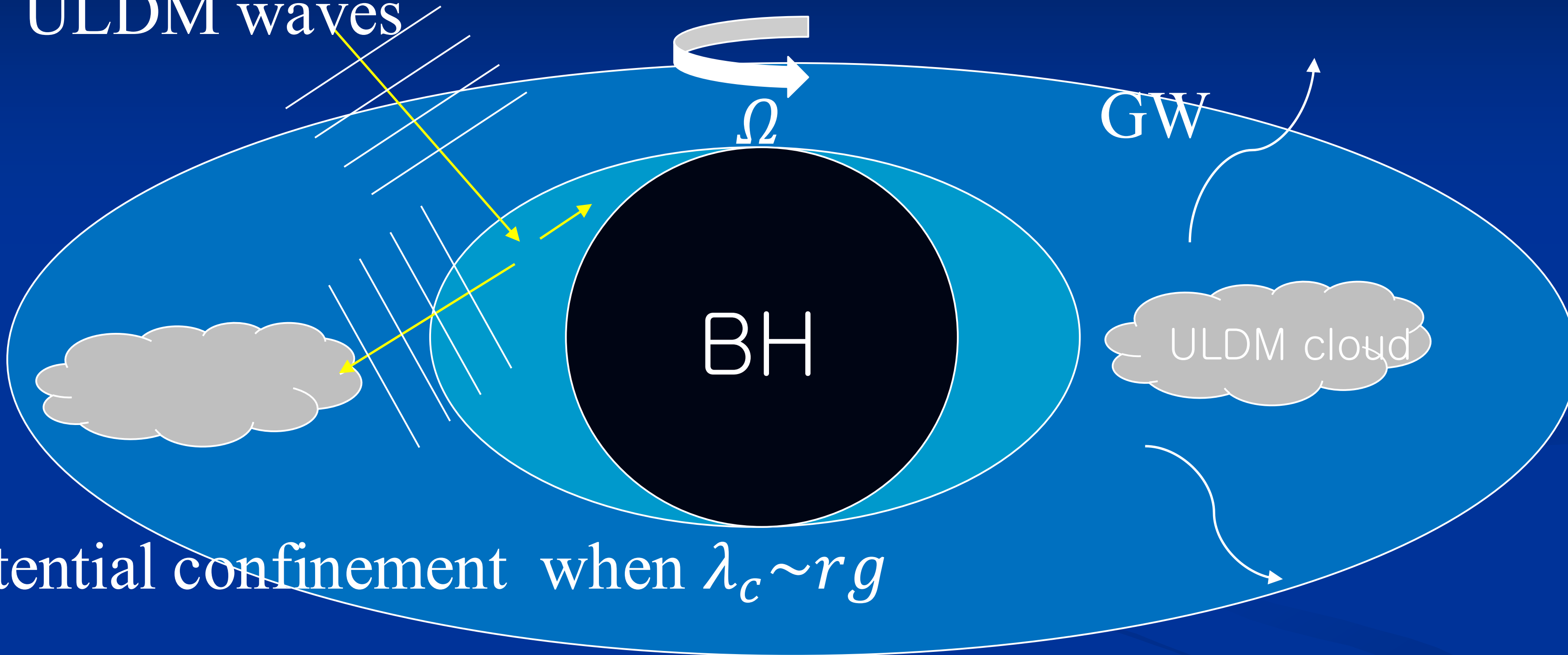
$$\psi_{nlm}(t,r) = R_{nl}(r)Y_{lm}(\theta,\phi)\mathrm{e}^{\mathrm{i}(\mu-\omega_{nlm})t}$$

gravitational fine structure constant

$$\alpha \equiv \frac{r_g}{\lambda_c} = \frac{GM\mu}{\hbar c} \approx 0.1\left(\frac{M}{10^{11}M_S}\right)\left(\frac{\mu}{10^{-22}\mathrm{eV}}\right) << 1$$

Occupation # is huge, presence of horizon

Superradiance
(Penrose process)



potential confinement when $\lambda_c \sim rg$

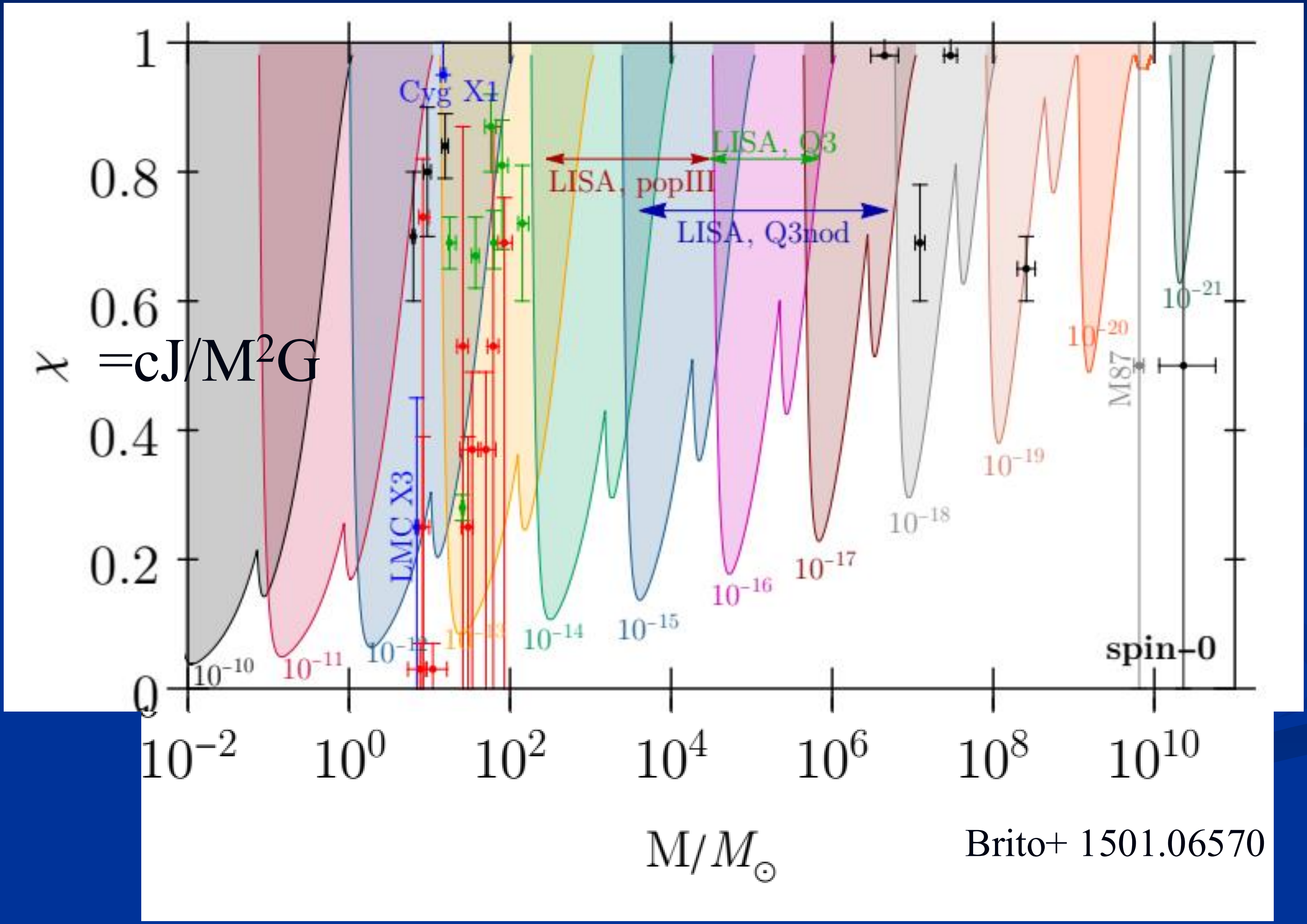
- growing mode if $\omega_{nlm} < m\Omega$ → BH spin decreases

  magnetic q. number

- can change GW patterns from a BH and BH binary

See Zhang & Yang 2018, 1908.10370, 2208.06408

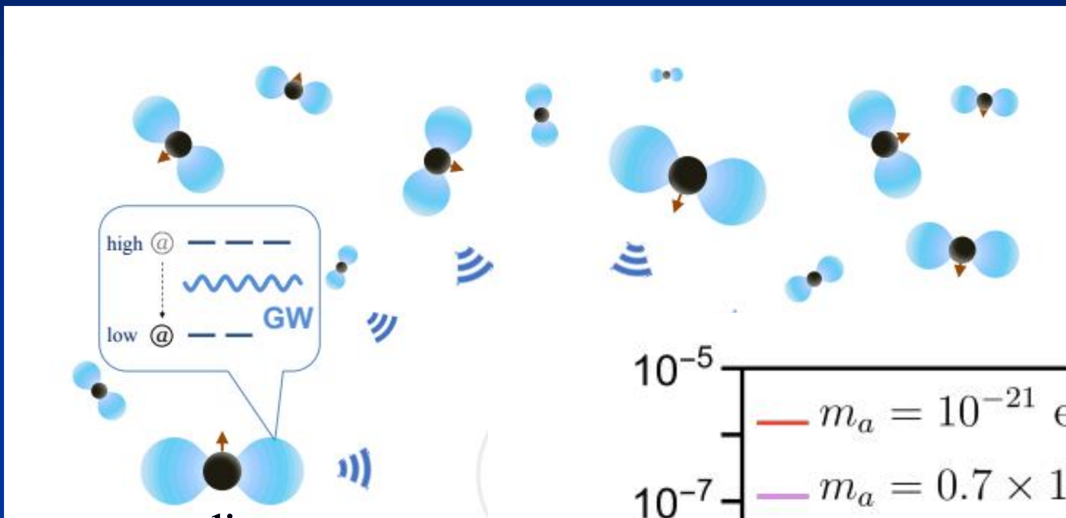
Bounds from BH-spin measurements (Regge plane)



Brito+ 1501.06570

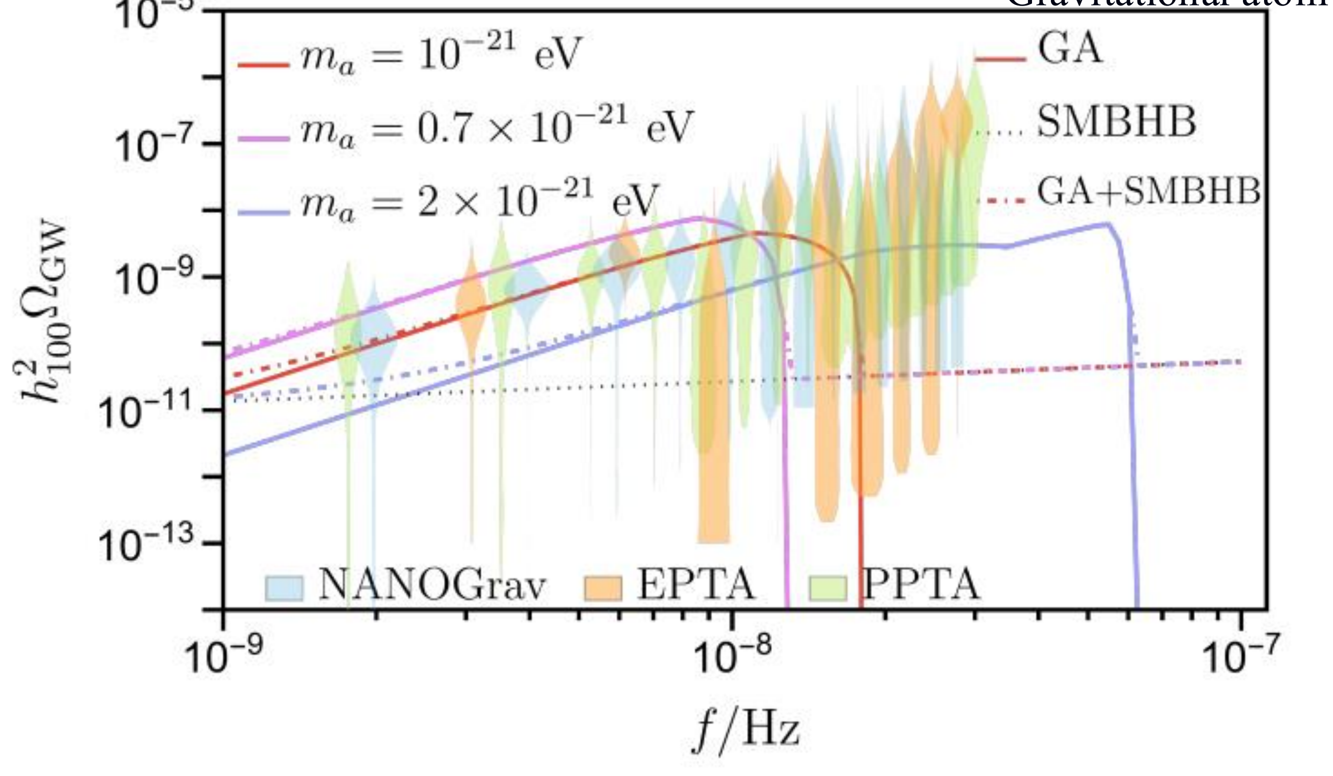
nano-Hertz stochastic GW background

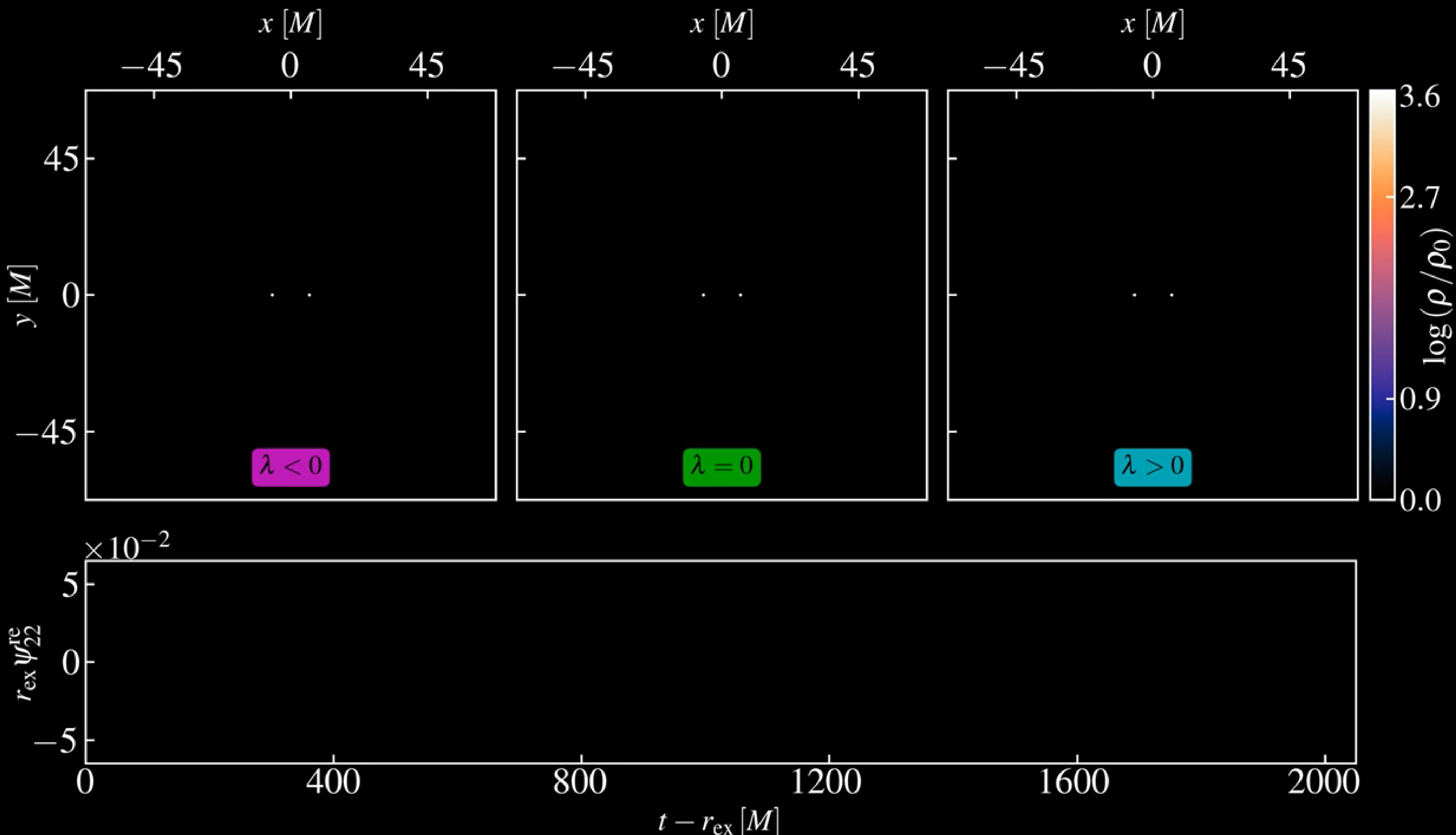
Yang+ 2306.17113



superradiance

Gravitational atom

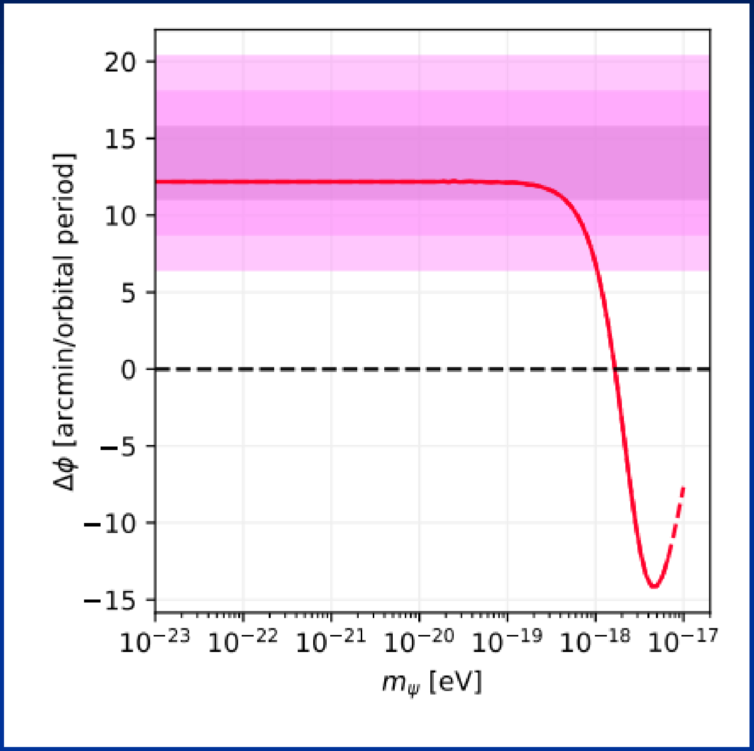


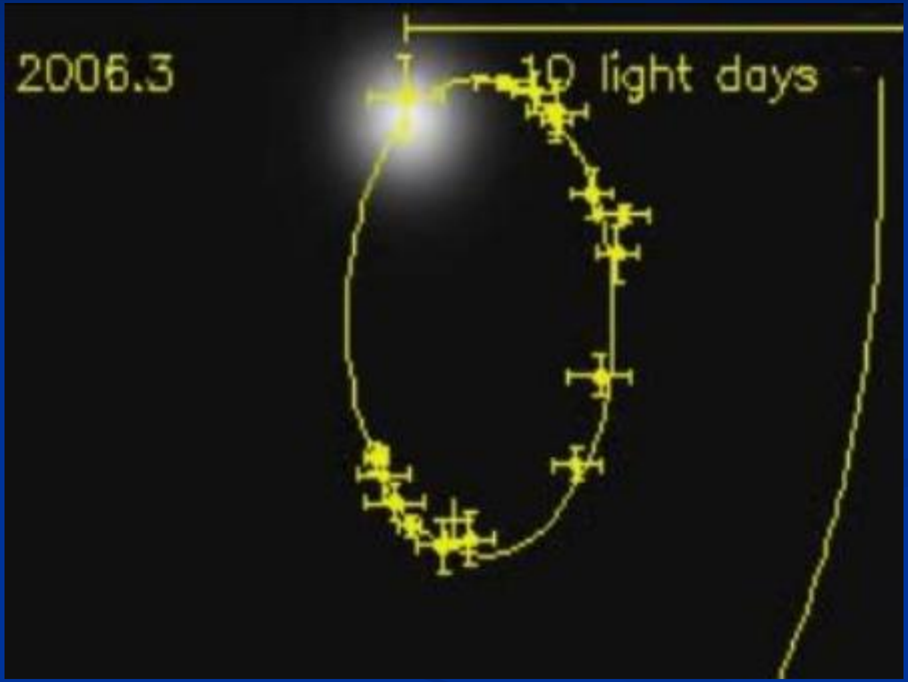


Aurrekoetxea+ PRD 2024

S2

Monica+ 2305.10242



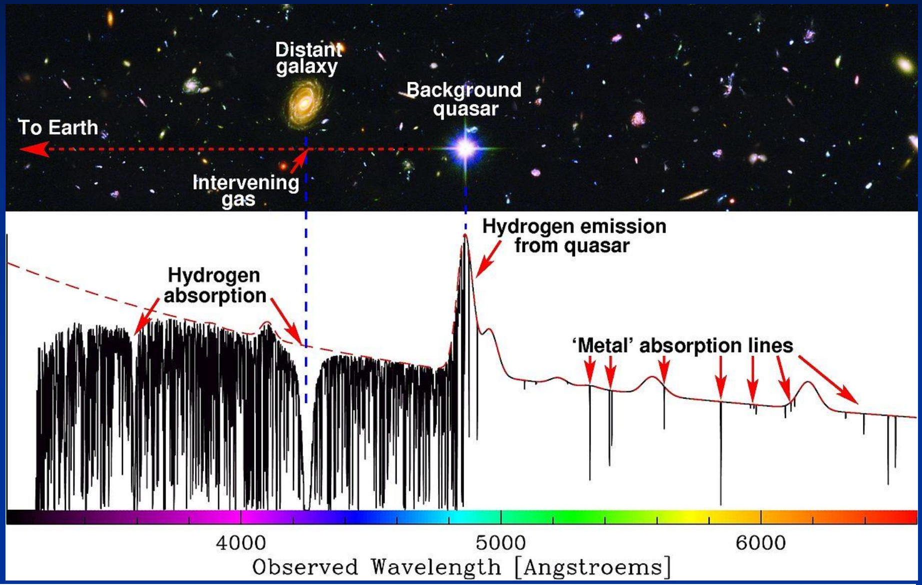


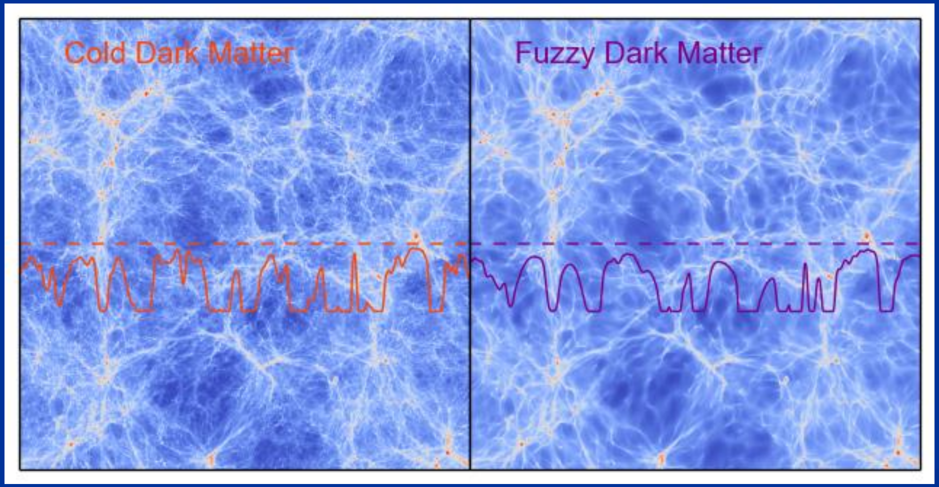
$$ds^2 = -F(r)dt^2 + \frac{1}{F(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$m_\psi \lesssim 3.2 \times 10^{-19}\mathrm{eV}$$

$$F(r) = \exp\left(\frac{4\pi\rho_c r_c^3}{91\alpha^8 r}\left(\left(\frac{r_c}{r}\right)^{13}\mathcal{F} - \frac{3003\pi\alpha^{\frac{13}{2}}}{2048}\right)\right) - \frac{2M}{r},$$

Lyman alpha tension?





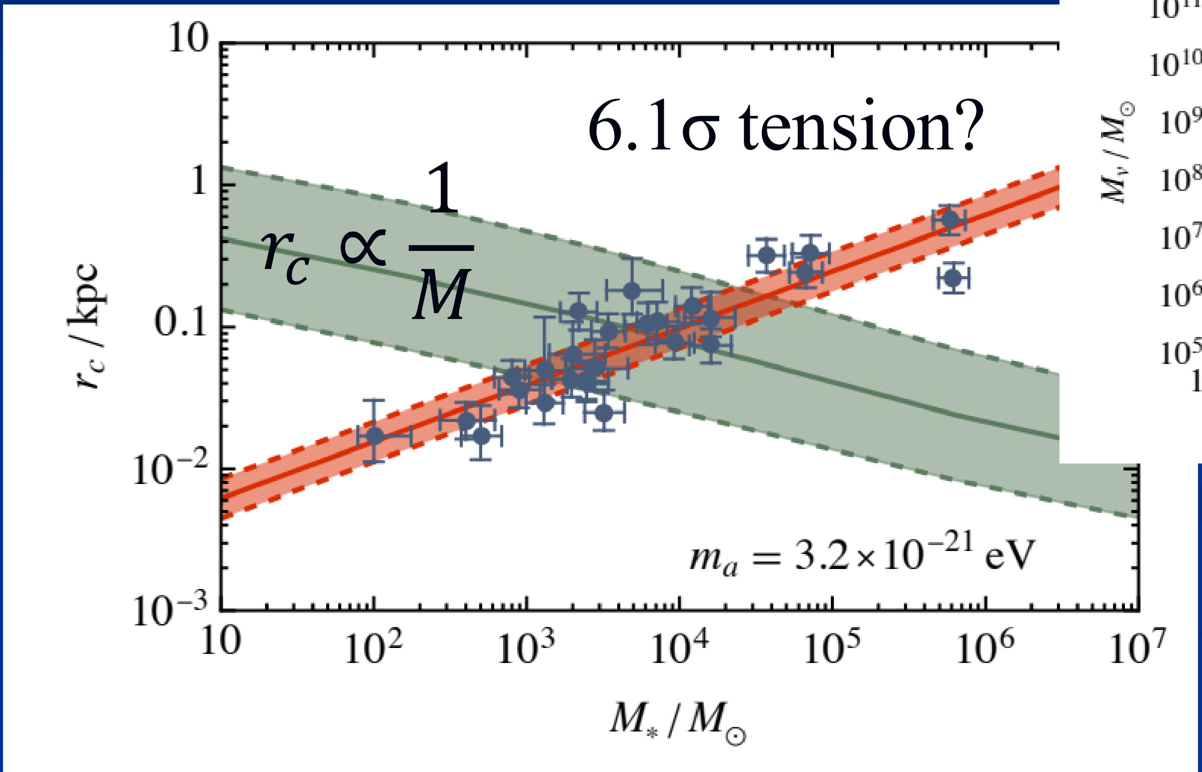
$m > 10^{-21}$eV

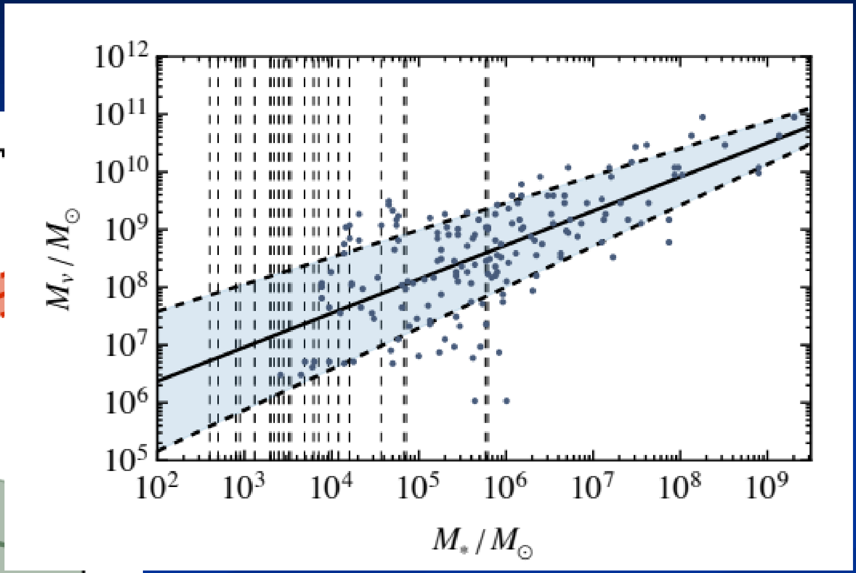
PRL 2017 (Irisic et al)

Hydrosimulation uncertainty is large

WDM (1, 3.3) keV ~ FDM (1, 20) x 10^{-22}eV

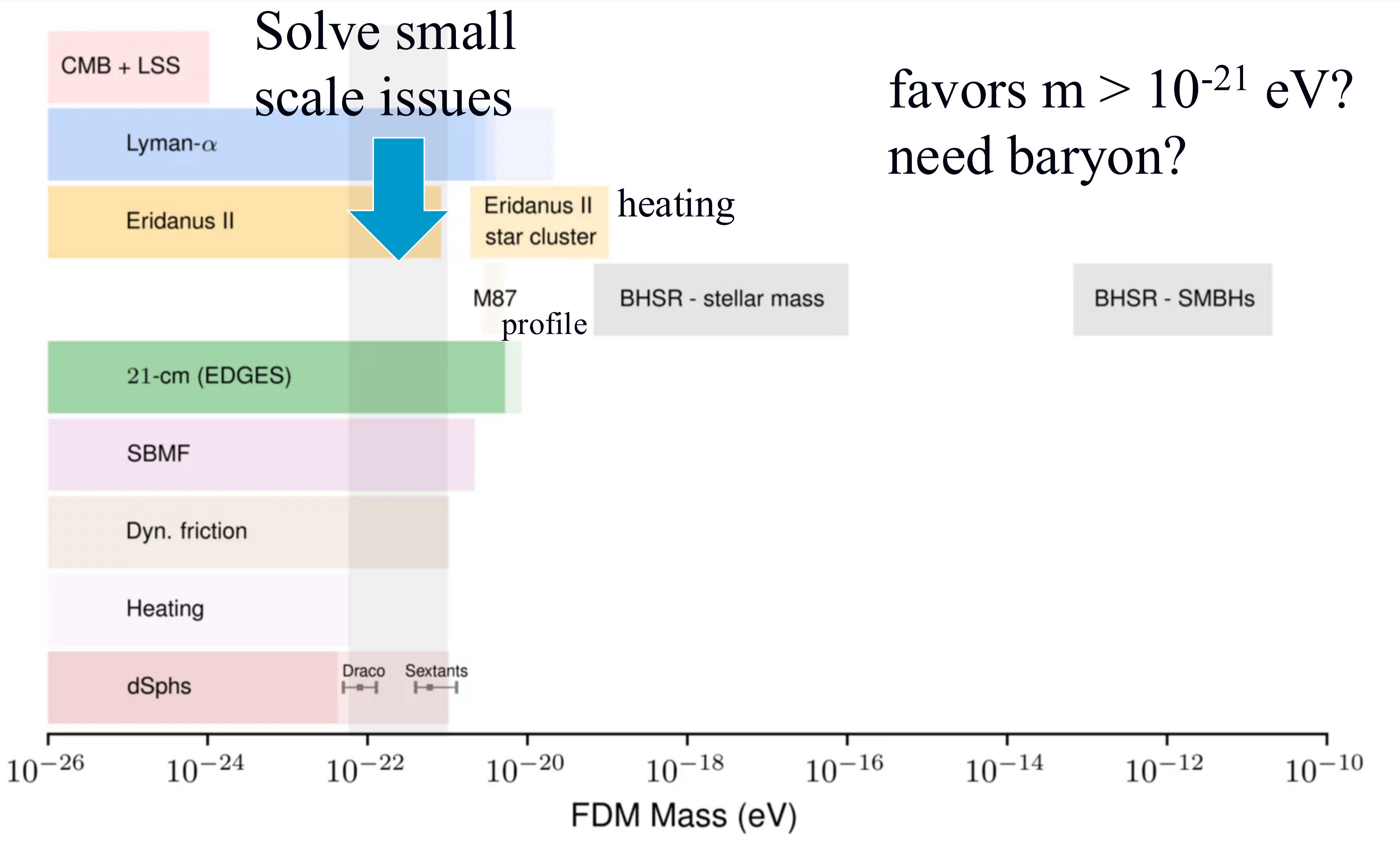
UFD constraints





Urrutia 2502.12030

Constraints on FDM (free ULDM) mass



https://www.mpa-garching.mpg.de/898225/hl202012?c=1004481

Merits of studying self-interacting UL DM

We can

- allow wider mass range

→avoid some tensions of FDM

- study direct, or indirect detection of ULDM
- calculate abundance
- understand particle model
- explain other mysteries like Hubble tension

& EW scale …

Constraints on particle SIDM

Positive observations	σ/m	$v_{\rm rel}$	Observation	Refs.
Cores in spiral galaxies (dwarf/LSB galaxies)	$\gtrsim 1\ \mathrm{cm}^2/\mathrm{g}$	$30 - 200\ \mathrm{km/s}$	Rotation curves	[102, 116]
Too-big-to-fail problem				
Milky Way	$\gtrsim 0.6\ \mathrm{cm}^2/\mathrm{g}$	$50\ \mathrm{km/s}$	Stellar dispersion	[110]
Local Group	$\gtrsim 0.5\ \mathrm{cm}^2/\mathrm{g}$	$50\ \mathrm{km/s}$	Stellar dispersion	[111]
Cores in clusters	$\sim 0.1\ \mathrm{cm}^2/\mathrm{g}$	$1500\ \mathrm{km/s}$	Stellar dispersion, lensing	[116, 126]
Abell 3827 subhalo merger	$\sim 1.5\ \mathrm{cm}^2/\mathrm{g}$	$1500\ \mathrm{km/s}$	DM-galaxy offset	[127]
Abell 520 cluster merger	$\sim 1\ \mathrm{cm}^2/\mathrm{g}$	$2000 - 3000\ \mathrm{km/s}$	DM-galaxy offset	[128, 129, 130]
Constraints				
Halo shapes/ellipticity	$\lesssim 1\ \mathrm{cm}^2/\mathrm{g}$	$1300\ \mathrm{km/s}$	Cluster lensing surveys	[95]
Substructure mergers	$\lesssim 2\ \mathrm{cm}^2/\mathrm{g}$	$\sim 500 - 4000\ \mathrm{km/s}$	DM-galaxy offset	[115, 131]
Merging clusters	$\lesssim \mathrm{few}\ \mathrm{cm}^2/\mathrm{g}$	$2000 - 4000\ \mathrm{km/s}$	Post-merger halo survival (Scattering depth $\tau < 1$)	Table II
Bullet Cluster	$\lesssim 0.7\ \mathrm{cm}^2/\mathrm{g}$	$4000\ \mathrm{km/s}$	Mass-to-light ratio	[106]

Tulin & Yu 1705.02358

Self-Interacting ULDM (SIULDM, SFDM)

Lee and Koh (PRD 53, hep-ph/9507385)

Galactic DM halo is described by coherent scalar field with self-interaction

Action $$S = \int \sqrt{-g}d^4x\left[\frac{-R}{16\pi G} - \frac{g^{\mu\nu}}{2}\phi^*_{;\mu}\phi_{;\nu} - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4\right]$$

Metric $$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega \ \ Spherical.$$

Field $$\phi(r,t) = (4\pi G)^{-\frac{1}{2}}\sigma(r)e^{-i\omega t}$$

$$\tilde{m} \equiv \frac{m}{\lambda^{\frac{1}{4}}}$$

Exact ground state (soliton)

$$\sigma_* = \sqrt{\frac{\gamma_0 \, \mathrm{Sin}(\sqrt{2}r_*)}{\sqrt{2}r_*}}$$

$$r_* = r\Lambda^{-1/2}$$

$\Lambda \equiv \frac{\lambda m_P^2}{4\pi m^2}$, $\Lambda >> 1$ (Newtonian & TF limit)

New constant length scale $RTF \approx \frac{\sqrt{\Lambda}}{m} = \sqrt{\frac{\pi\hbar^3\lambda}{8cGm^4}} \sim \tilde{m}^{-2}$

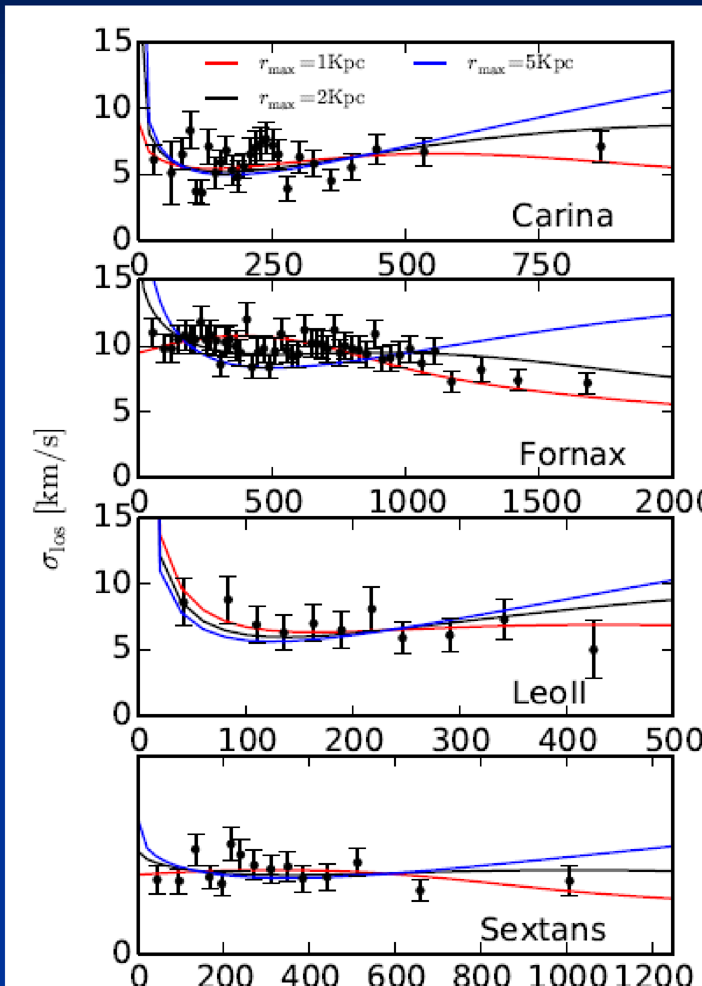
& mass scale $M_{\max} = \sqrt{\Lambda}\frac{m_P^2}{m}$

- Even tiny self-interaction drastically changes the scales!

→ allows wider range for m to fit observations

- $\tilde{m}$ determines all scales

Alberto Diez-Tejedor etal PRD 2014



$$\lambda^{1/2}\left(\frac{m_P}{m}\right)^2 \geq 10^{50} \rightarrow \left(\frac{m_P}{10^{25}}\right) \approx 1221 eV \geq \frac{m}{\lambda^{1/4}}$$

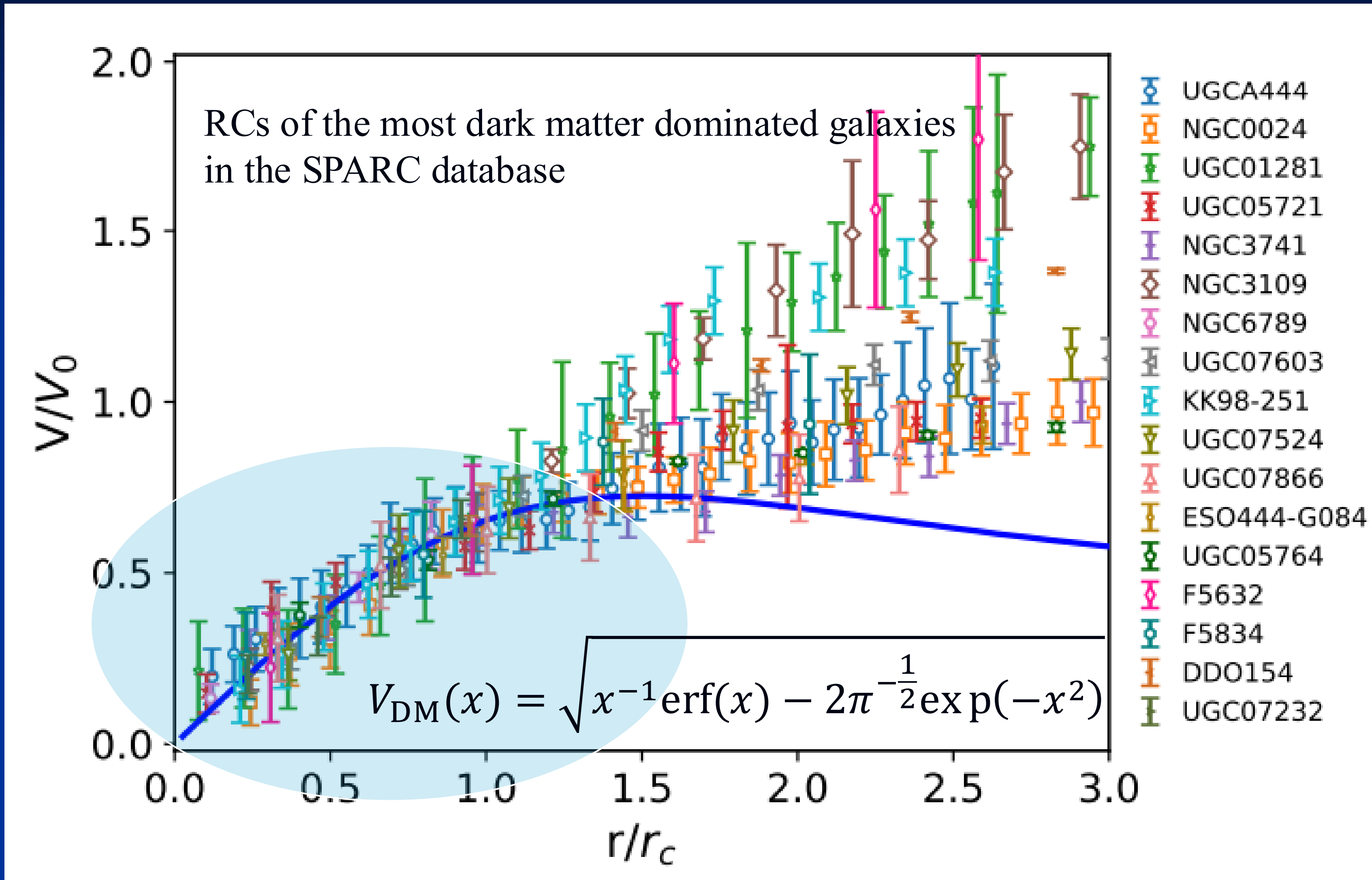
$$\rho = \sigma^2 = \rho_0 \frac{\sin(\pi r / r_{\max})}{\pi r / r_{\max}}$$

$$r_{\max} \equiv \sqrt{\frac{\pi^2 \Lambda}{2}} \sim 49 \left(\frac{\lambda^{1/4}}{m / eV}\right)^2 kpc$$

dSphs $\quad r_{\max} \sim 1 kpc \rightarrow m / \lambda^{1/4} \sim 7 eV$

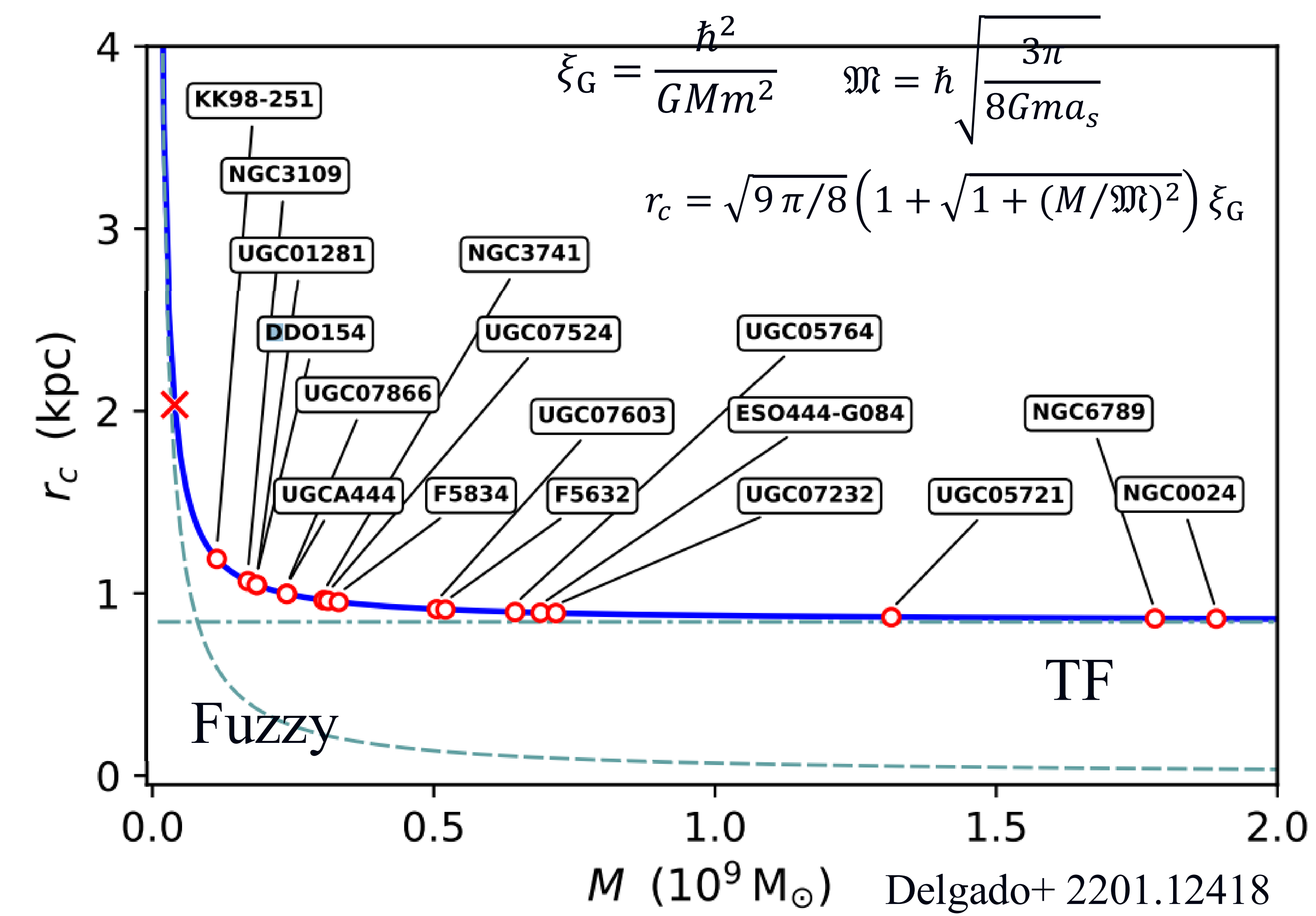
$$3\text{ eV} < \tilde{m} = \frac{m}{\lambda^{\frac{1}{4}}} < 7\text{ eV}$$

Universal RC with SIULDM



$x = r/r_c$

Delgado+ 2201.12418



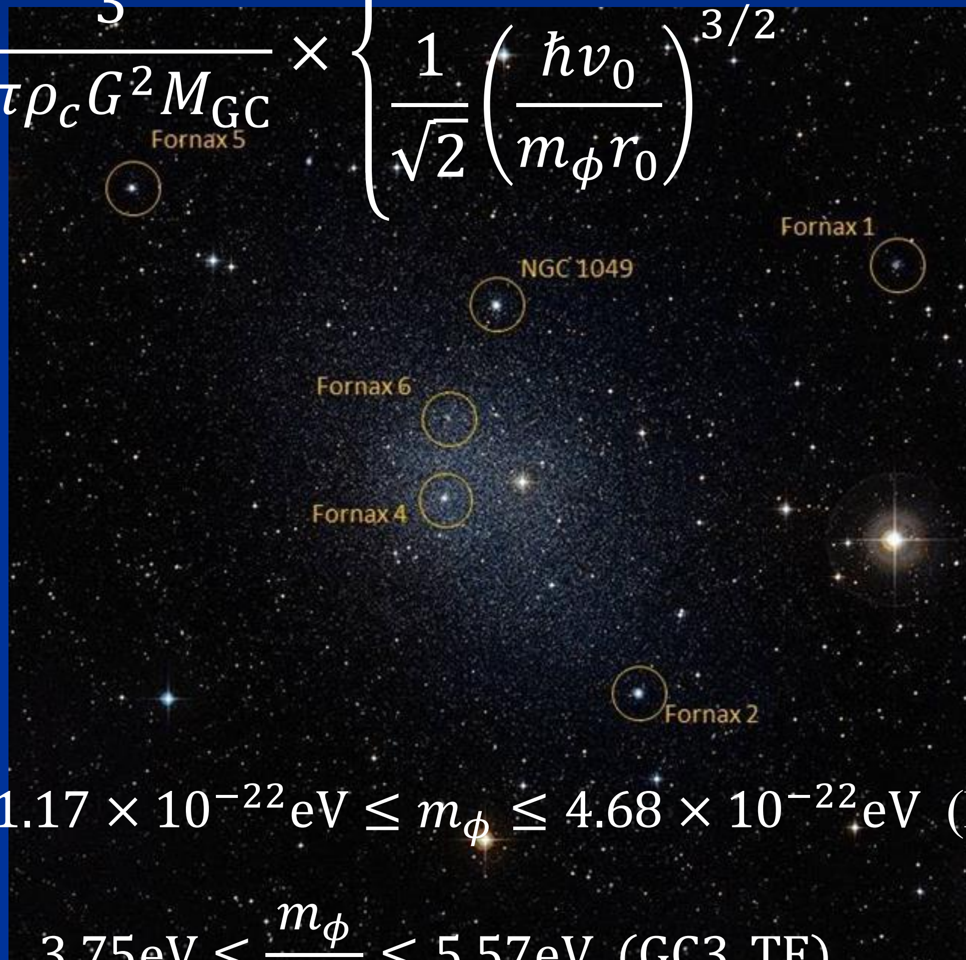
best fit $m \simeq 2.2 \times 10^{-22}$eV, $a_s = \frac{\lambda}{8\pi m} \simeq 7.8 \times 10^{-77}$ meter

$\rightarrow \tilde{m} = 5.72$ eV

Timing problem of GC

$$\bar{\tau} = \frac{3}{4\pi\rho_c G^2 M_{\rm GC}} \times \begin{cases} c_s^3 & \textit{(Thomas Fermi limit)} \\ \frac{1}{\sqrt{2}}\left(\frac{\hbar v_0}{m_\phi r_0}\right)^{3/2} & \textit{(FDM limit).} \end{cases}$$

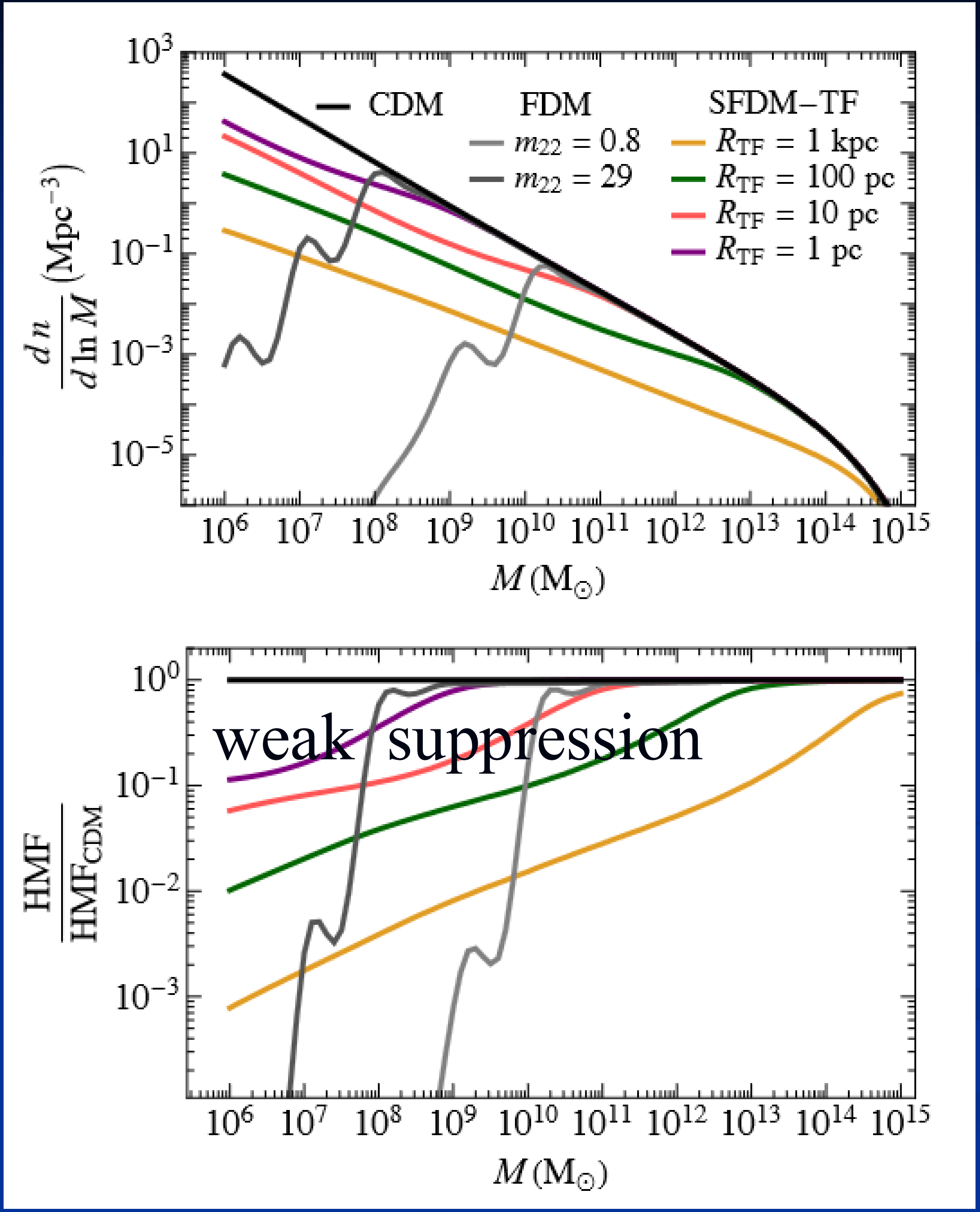
Koo & Lee
JCAP 2026



$$1.17 \times 10^{-22}\text{eV} \le m_\phi \le 4.68 \times 10^{-22}\text{eV} \ \ \text{(FDM)}$$

$$3.75\text{eV} \le \frac{m_\phi}{\lambda^{1/4}} \le 5.57\text{eV} \ \ \text{(GC3, TF)}$$

Shapiro+ 2106.13244

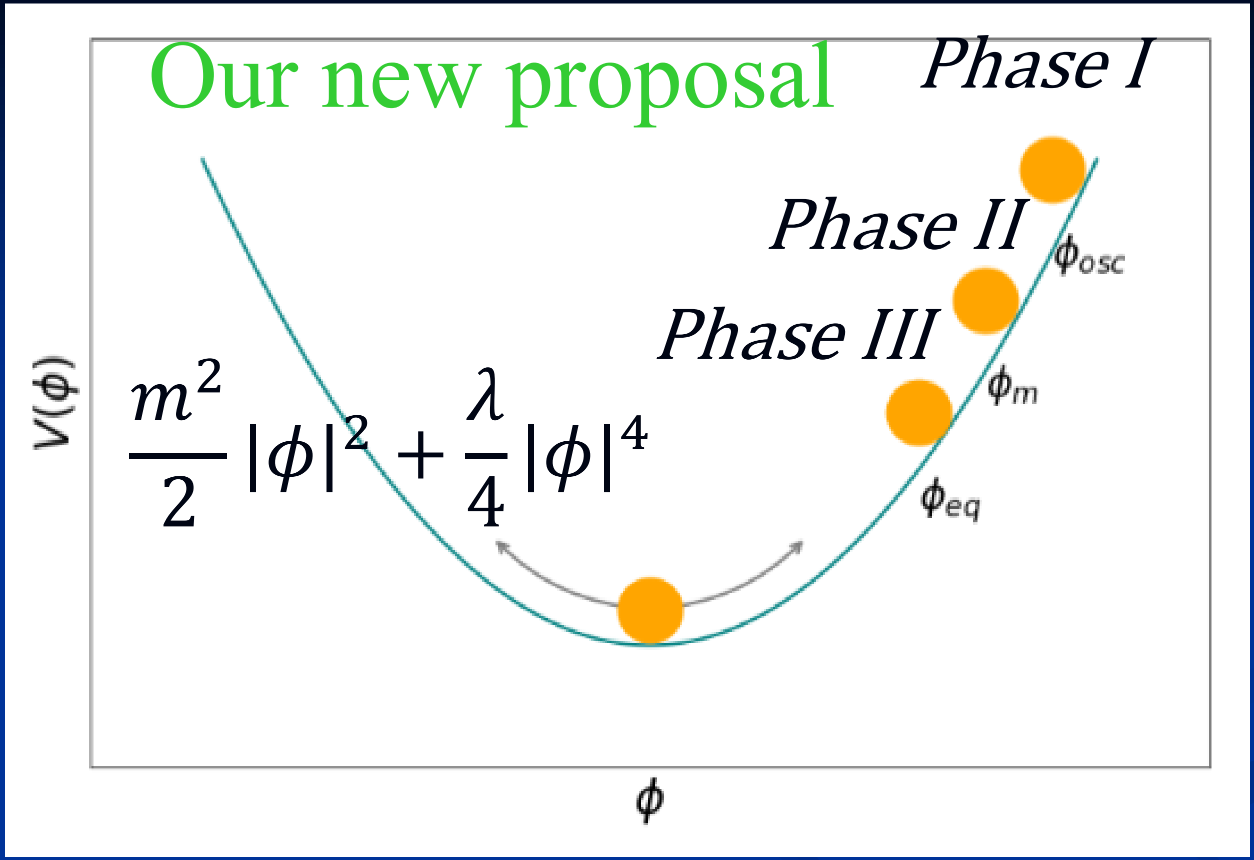


λ_J *fixed*

$M_J \propto a^{-3}$

weak suppression
→ weak missing satellites

→ may avoid Lyman alpha tension



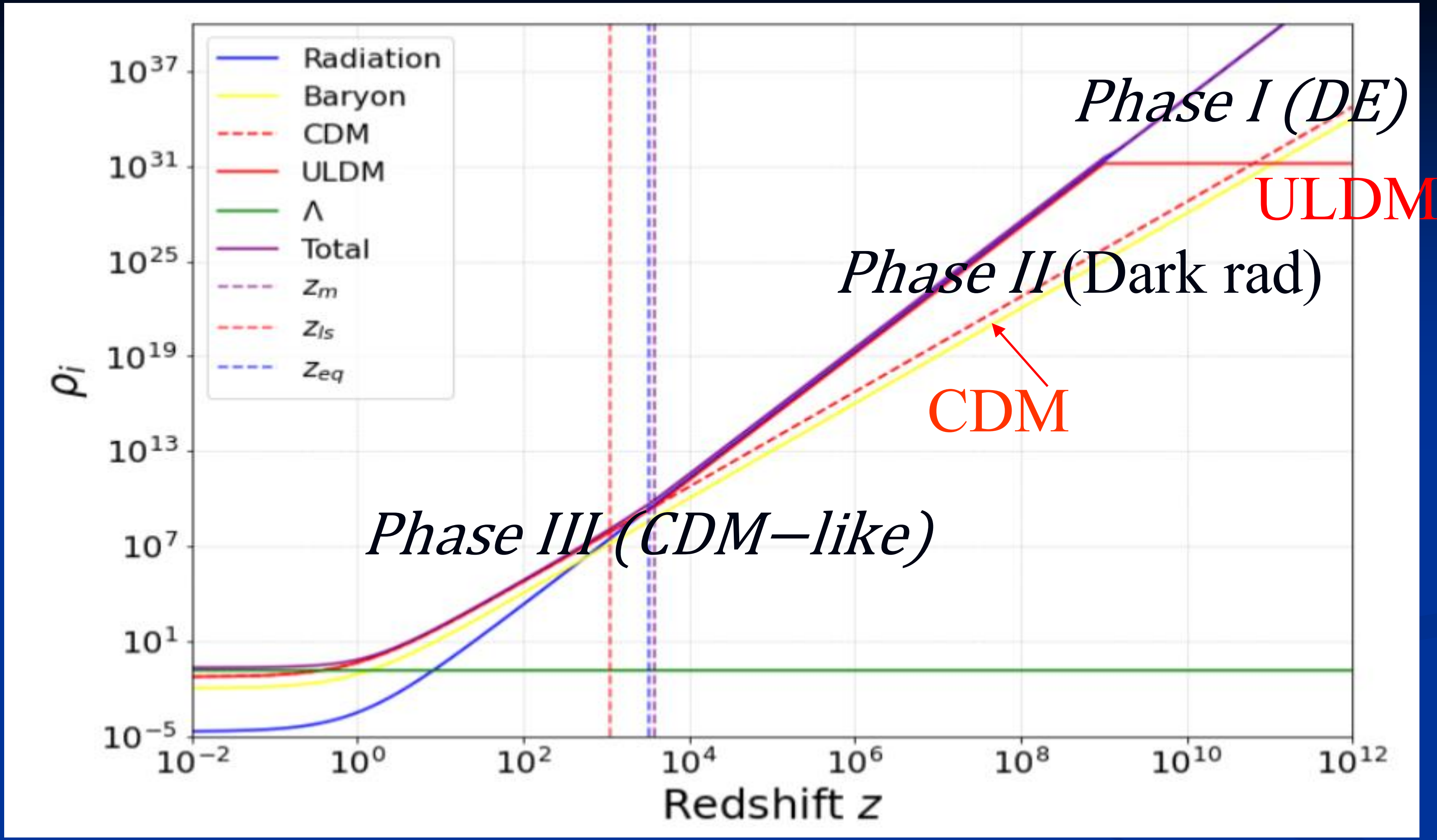
JLee
2502.11568

$V(\phi) \propto \phi^n$ decays with an equation of state $w = \frac{(n-2)}{(n+2)}$

$$\begin{cases} DE-like\ (slow-roll)\ for\ \ z_{osc} < z\ (Phase\ I) \\ Dark\ rad\text{–}like\ (\phi^4 > \phi^2)\ or\ \ z_m < z < z_{osc}\ (Phase\ II) \\ \text{CDM}\ \ (\phi^4 < \phi^2)\ \text{or}\ \ z < z_m\ (Phase\ III) \end{cases}$$

or stiff

SIULDM alone is enough!



→ SIULDM acts as a natural extra radiation comp. just before ls

→ We don't need exotic matter or fine tuning?

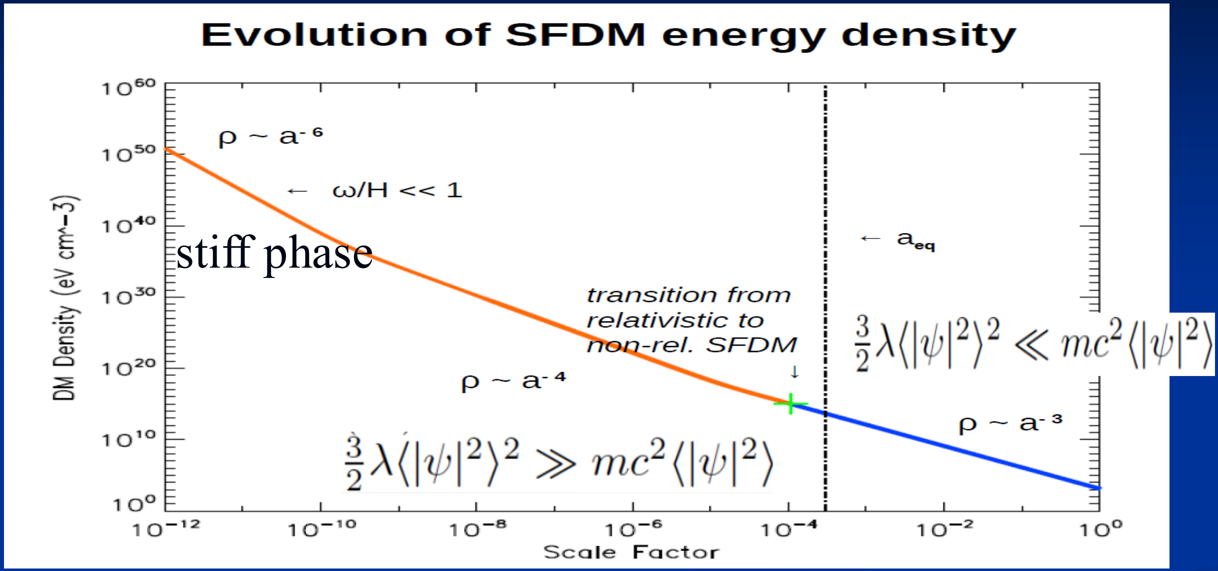
Li etal 1310.6061

Cosmological evolution

$$V(\psi) = \frac{1}{2} mc^2 |\psi|^2 + \frac{\lambda}{2} |\psi|^4$$

$$\langle \bar{w} \rangle \equiv \frac{\langle \bar{p} \rangle}{\langle \bar{\rho} \rangle} = \frac{1}{3} \left[\frac{1}{1 + \frac{2mc^2}{3\lambda \langle |\psi|^2 \rangle}} \right]$$

Time varying Eq. of state



1) z_{eq}=3390

$$\frac{\lambda}{(mc^2)^2} \leq 4 \times 10^{-17} \text{ eV}^{-1} \text{ cm}^3.$$

$$\Rightarrow m^4/\lambda > 4.7 \times 10^{-7} eV^4$$

2) Big Bang Nucleosynthesis

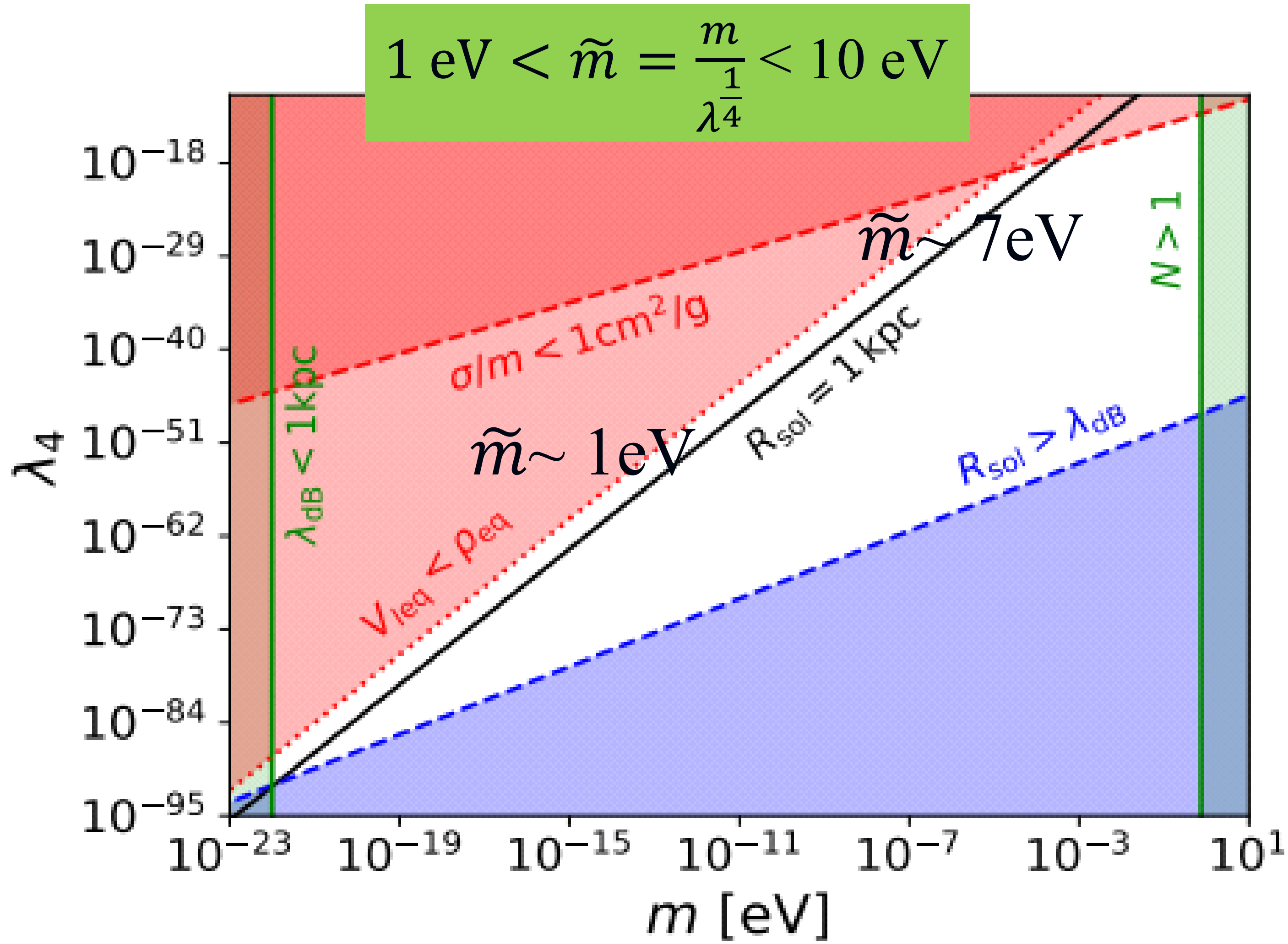
$$\frac{\Delta N_{eff}}{N_{eff,\text{standard}}} = \frac{3.71^{+0.47}_{-0.45} - 3.046}{3.046} = \frac{\rho_{SFDM}}{\rho_\nu}$$

$$9.5 \times 10^{-19} \text{ eV}^{-1} \text{ cm}^3 \leq \lambda/(\text{mc}^2)^2 \leq 1.5 \times 10^{-16} \text{ eV}^{-1} \text{ cm}^3$$

$$\rightarrow 2.6eV \leq \frac{m}{\lambda^{1/4}} \leq 9.5eV$$

cosmological constraints

Garcia + 2304.10221



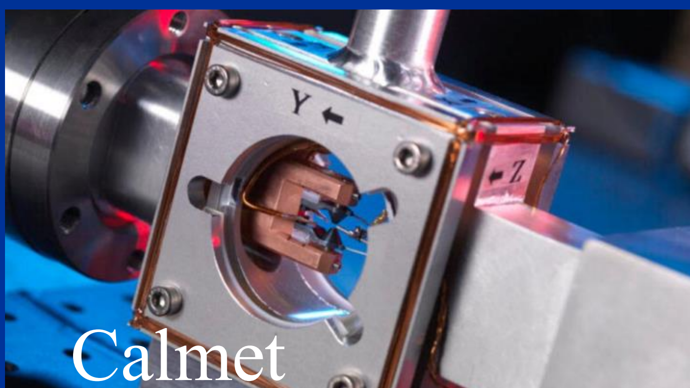
Detection of interacting ULDM

dilatonic coupling

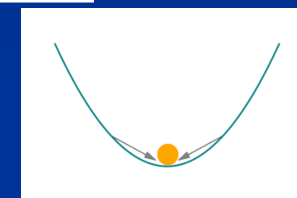
$$\mathcal{L} \supset \varphi \frac{d_e}{4\mu_0} F_{\mu\nu} F^{\mu\nu} ,$$

Using atomic clocks to detect ULDM by mimicking time variations of fundamental constants

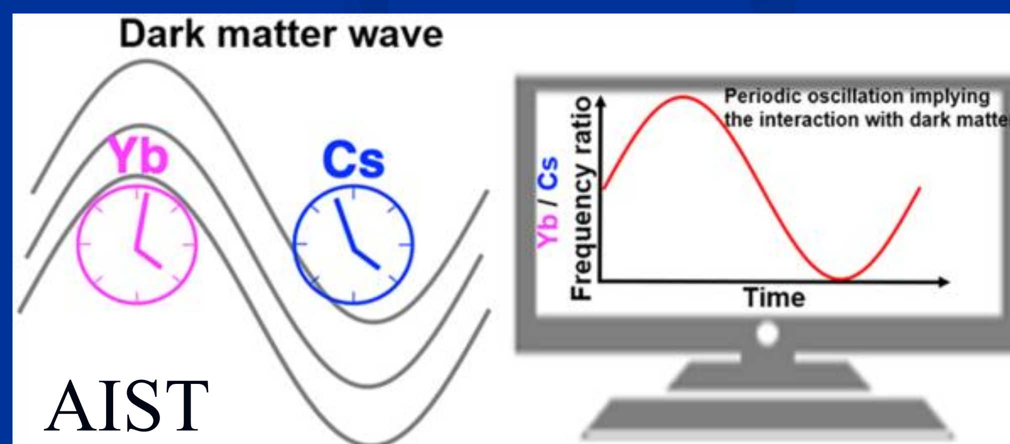
$$\alpha(t) \approx \alpha \left[1 + d_e \varphi_0 \cos(\omega t + \delta)\right]$$



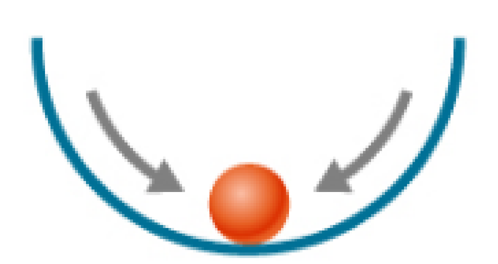
Oscillation of fine structure constant



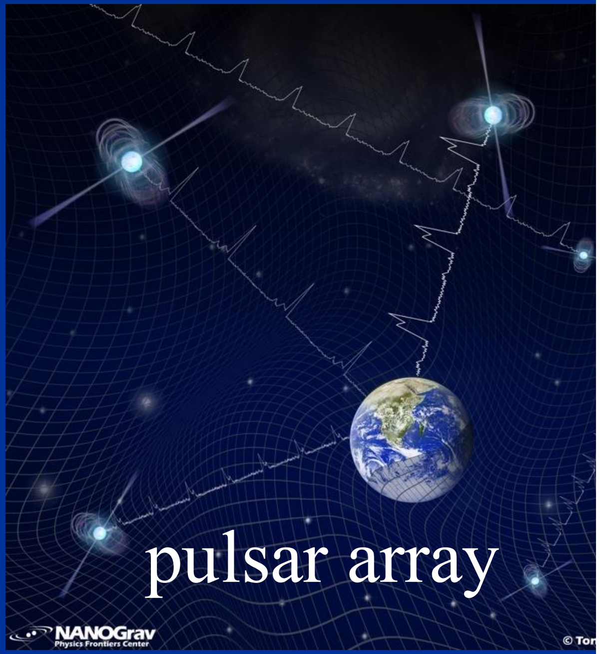
Due to nuclear and atomic structure Yb and Cs have different frequency dependency on α (special relativistic effects)

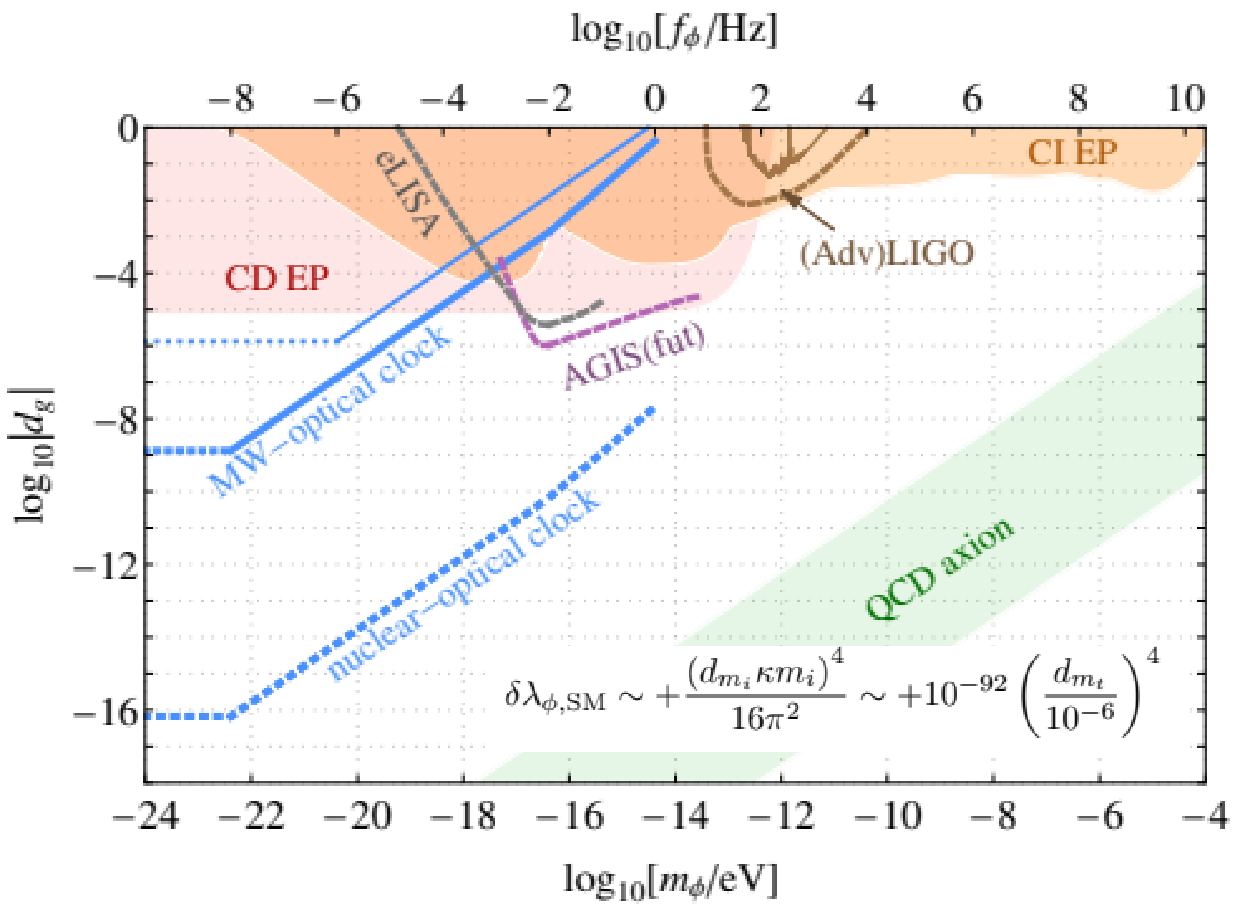


$$\omega = \frac{1}{2.5 months}\frac{m}{10^{-22}eV}$$



$$\frac{1}{4g^2}F_{\mu\nu}F^{\mu\nu} + \frac{\varphi d_e}{4g^2}F_{\mu\nu}F^{\mu\nu} \Longrightarrow \alpha(t) \approx \alpha\left[1 + d_e\varphi_0\cos(\omega t + \delta)\right]$$





Arvanitaki+1405.2925

Direct detection

* The common low-energy effective interaction is

$$\mathcal{L}_\phi = \kappa\phi\mathcal{O}_{\rm SM},$$

where $\kappa \equiv \sqrt{4\pi}/M_{\rm Pl}$ and

$$\mathcal{O}_{\rm SM} = d_e\frac{F_{\mu\nu}F^{\mu\nu}}{4} - d_g\frac{\beta_3}{2g_3}G^A_{\mu\nu}G^{A\mu\nu} - d_{me}m_e\bar{e}e - \sum_i d_{m_i}m_i\bar{\psi}_i\psi_i$$

1) Atomic, Molecular, Nuclear Clocks

Detect tiny oscillations of the fine-structure constant α and electron-proton mass ratio.

Frequency ratios of different clocks vary as $\phi(t) \propto \cos(m_\phi t)$.

Effective Lagrangian:

$$\mathcal{L} \supset \kappa\phi\left[d_e\frac{F_{\mu\nu}F^{\mu\nu}}{4} + d_{me}m_e\bar{e}e + d_{\hat{m}}(m_u\bar{u}u + m_d\bar{d}d)\right]$$

2) Optical Interferometers (including GW detectors)

- Sensitive to oscillations of α that change optical path length or refractive index.
- Effective Lagrangian:

$$\mathcal{L} \supset \kappa\phi d_e\frac{F_{\mu\nu}F^{\mu\nu}}{4}.$$

Atom Interferometers

- Detect variations of atomic mass m_A and gravitational potential.
- Effective Lagrangian: $$\mathcal{L} \supset \kappa\phi\left[d_{\hat{m}}\sum_q m_q\bar{q}q + d_g\frac{\beta_3}{2g_3}G^A_{\mu\nu}G^{A\mu\nu}\right].$$

Torsion Balances and Equivalence Principle Tests

- Search for fifth forces and tiny composition-dependent accelerations.
- Effective Lagrangian:

$$\mathcal{L} \supset \kappa\phi d_g \frac{\beta_3}{2g_3} G^A_{\mu\nu} G^{A\mu\nu}$$

LC Oscillators and Mechanical Resonators

- Look for oscillations of electrical or mechanical resonance frequencies.
- Effective Lagrangian:

$$\mathcal{L} \supset \kappa\phi d_e \frac{F_{\mu\nu}F^{\mu\nu}}{4}$$

Haloscopes (cavity, plasma, dielectric)

- Probe scalar or axion-like couplings to photons.
- Scalar case:

$$\mathcal{L} \supset \kappa\phi d_e \frac{F_{\mu\nu}F^{\mu\nu}}{4}$$

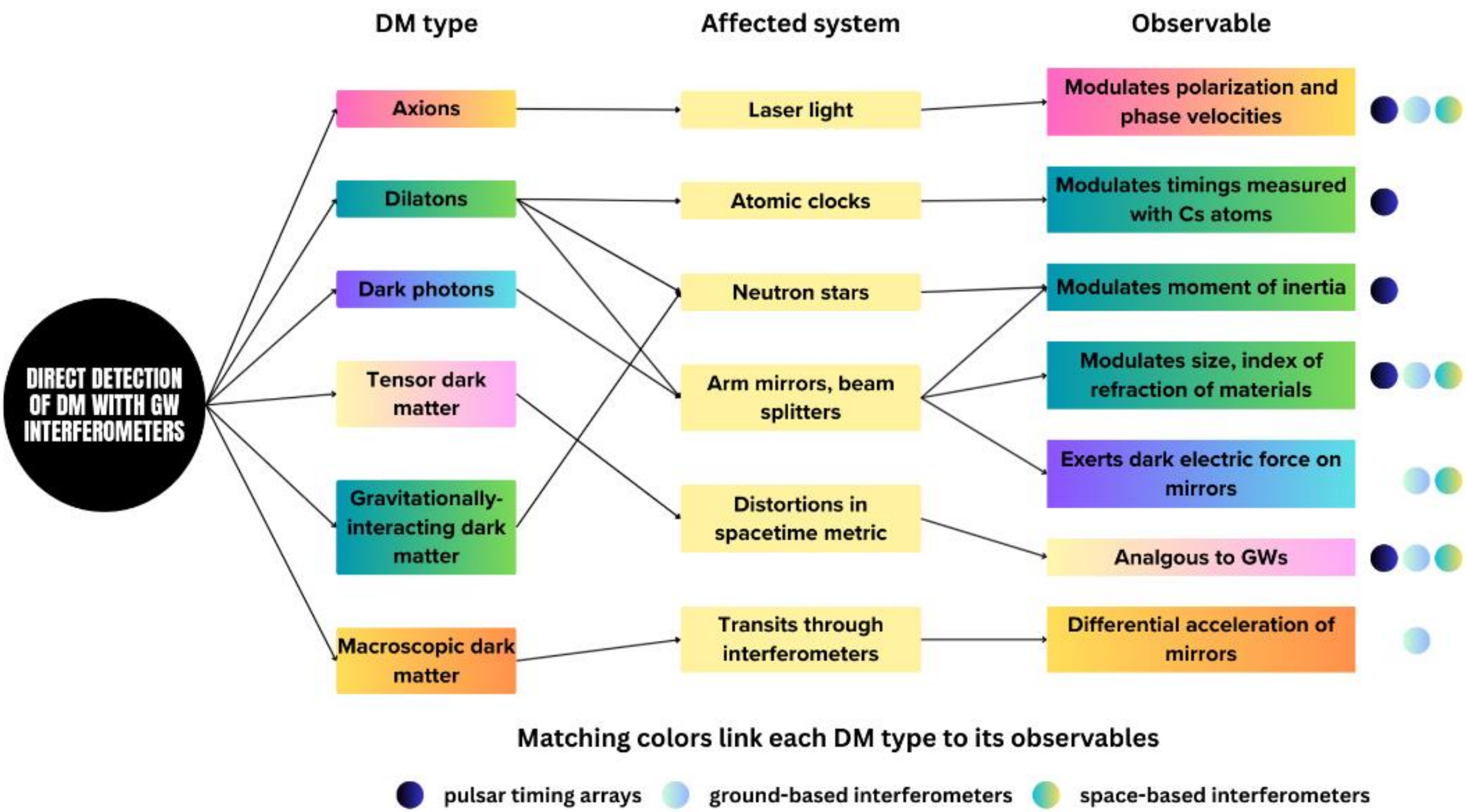
- Pseudoscalar (axion-like) case:

$$\mathcal{L} \supset \frac{g_{\phi\gamma\gamma}}{4} \phi F_{\mu\nu}\tilde{F}^{\mu\nu}.$$

Spin-Based Sensors

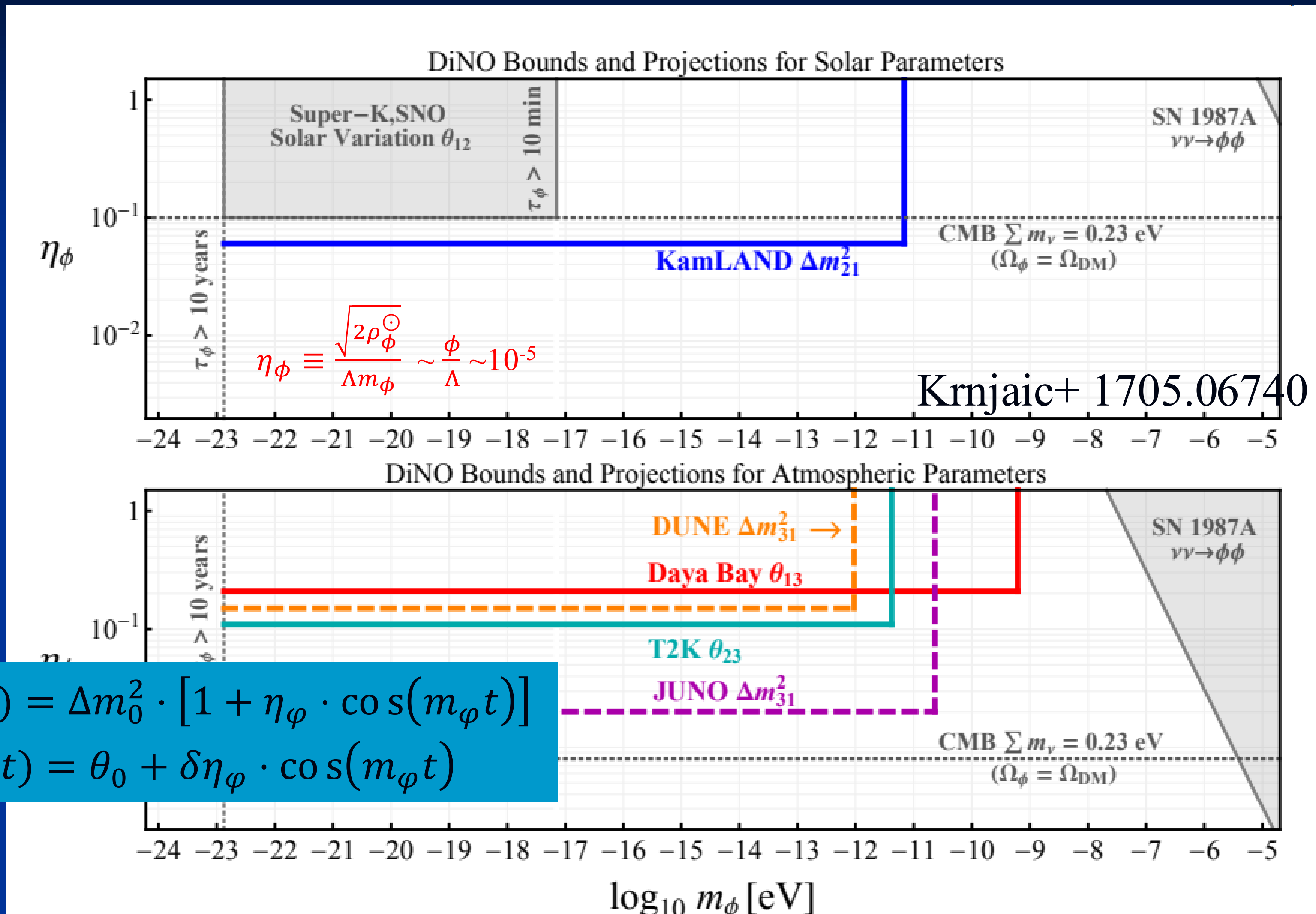
- Measure shifts in Larmor precession of electron or nuclear spins.
- Effective Lagrangian:

$$\mathcal{L} \supset \kappa\phi d_{me} m_e \bar{e}e.$$



Miller 2503.02607

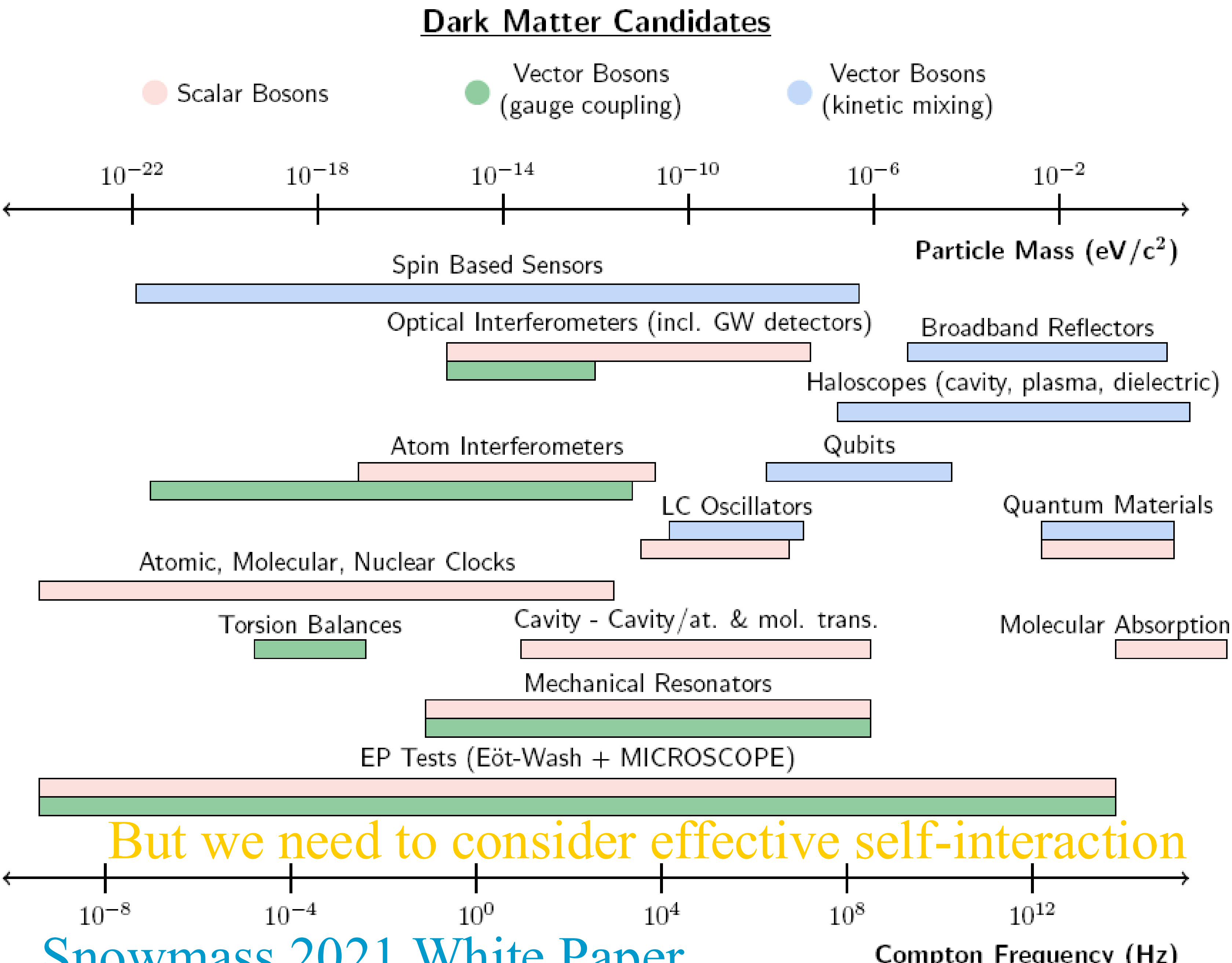
neutrino oscillation with ULDM



$$\Delta m^2(t) = \Delta m_0^2 \cdot \left[1 + \eta_\varphi \cdot \cos\left(m_\varphi t\right)\right]$$

$$\theta(t) = \theta_0 + \delta\eta_\varphi \cdot \cos\left(m_\varphi t\right)$$

$$\mathcal{L}_{eff} = -m_\nu \left(1 + y_1 \frac{\phi}{\Lambda}\right) \nu\nu, \qquad \phi(x,t) \simeq \frac{\sqrt{2\rho_\phi^\odot}}{m_\phi} \cos\left[m_\phi (t - \vec{v}\cdot\vec{x})\right]$$



But we need to consider effective self-interaction

Snowmass 2021 White Paper

Conclusions

ULDM with m~10^{-21} eV or

self-interacting ULDM with $\frac{m}{\lambda^{1/4}} \sim 1eV$

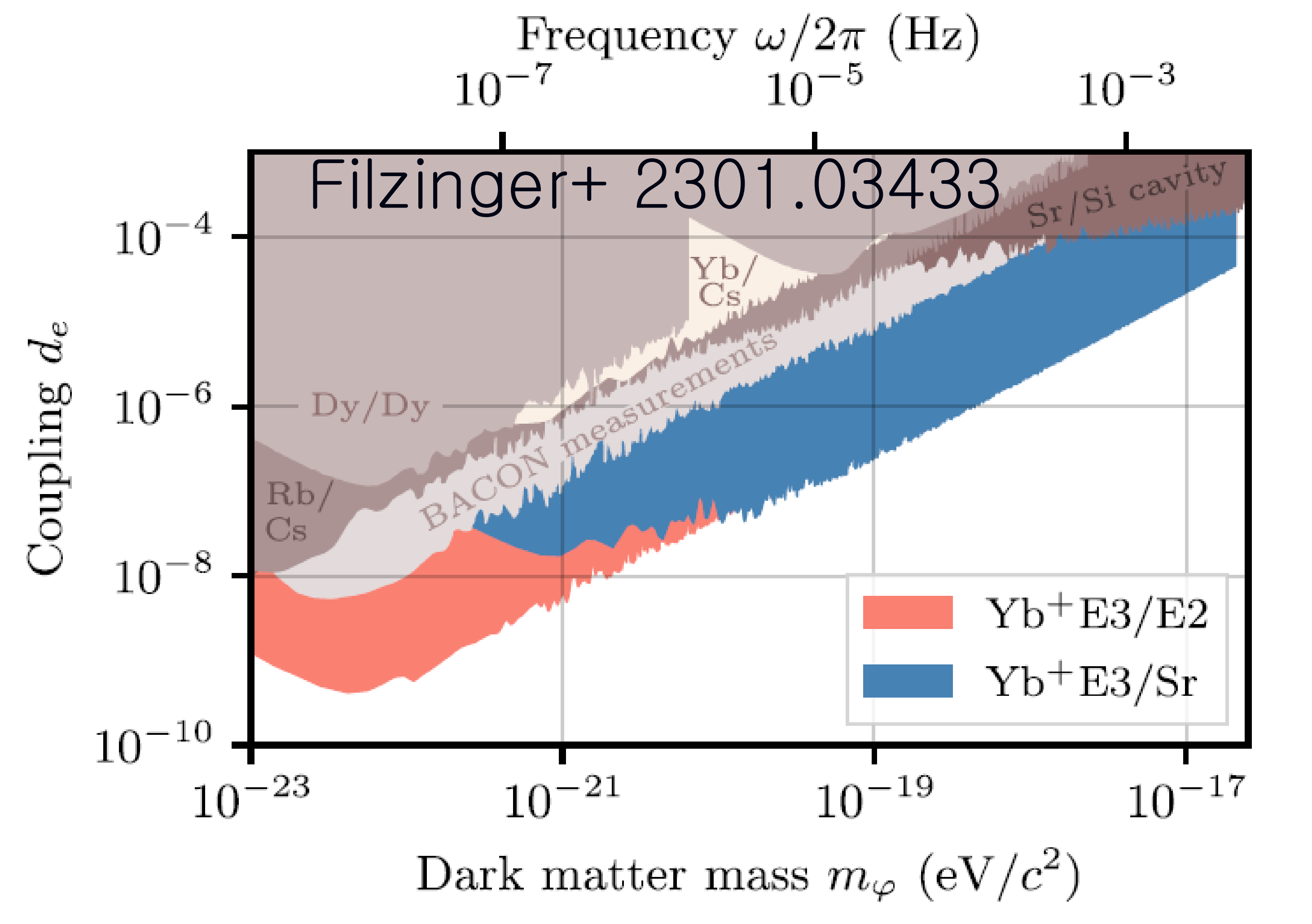
seems to be a viable alternative to CDM

→ might solve many mysteries of cosmology and astrophysics.
→ can be detected soon!

detection

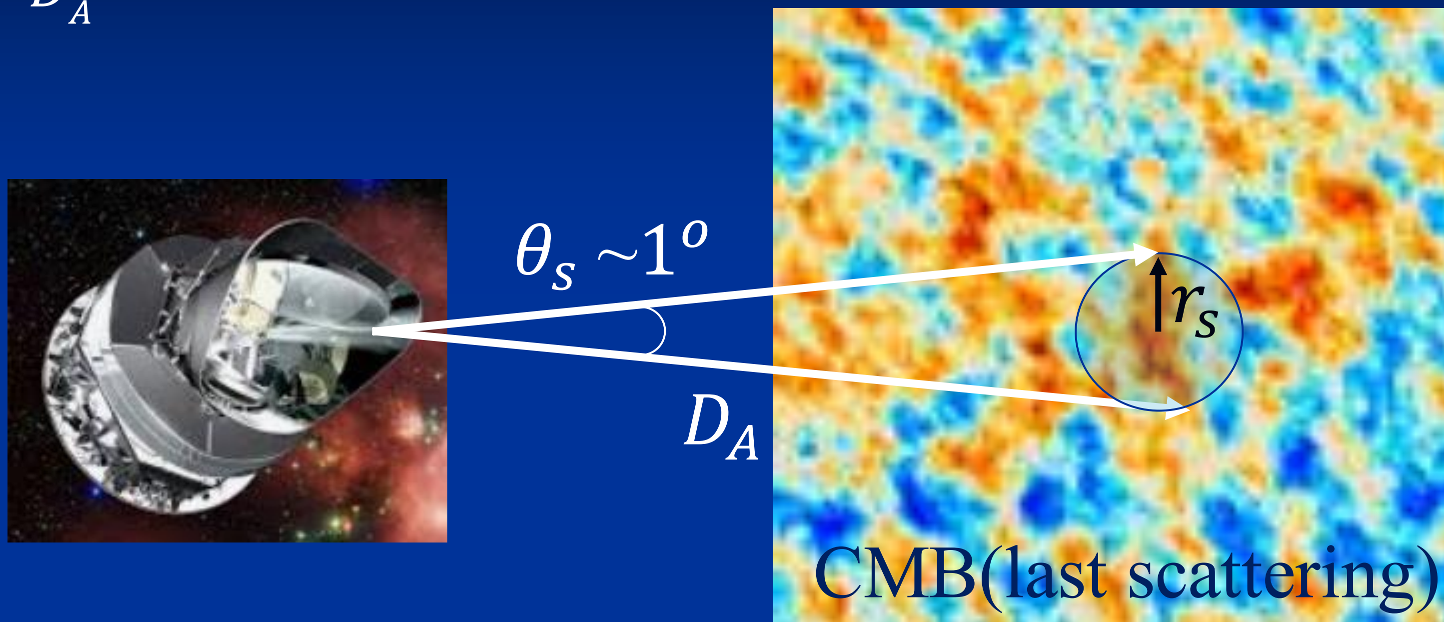
Oscillation of fine structure constant

$$\alpha(t) \approx \alpha\left[1 + d_e \varphi_0 \cos(\omega t + \delta)\right]$$



H_0 estimation from CMB

$\theta_s = \frac{r_s}{D_A}$ is a standard ruler with 0.04% precision. (fixed)



Sound Horizon $\quad r_s = \frac{c}{\sqrt{3}H_{\mathrm{ls}}} \int_{z_{\mathrm{ls}}}^{\infty} \frac{dz}{[\rho(z)/\rho(z_{\mathrm{ls}})]^{1/2}(1+R(z))^{1/2}}$

Angular distance $\quad \boxed{D_A = \frac{c}{H_0} \int_0^{z_{ls}} \frac{dz}{[\rho(z)/\rho_0]^{1/2}} \sim \frac{1}{H_0}}$

r_s & D_A depend on expansion history (matter contents $\rho(z)$)

$$H_0 = \theta_s H_{ls} \frac{\int_{z_{ls}}^{0} \frac{c\,dz}{[\rho(z)/\rho_0]^{1/2}}}{\int_{\infty}^{z_{ls}} \frac{c_s(z)\,dz}{[\rho(z)/\rho(z_{ls})]^{1/2}}}$$

late time (decrease ρ(z))

early time (increase ρ(z))

To increase H_0 we can ($\theta_s = \frac{r_s}{D_A}\ fixed$)

1) in early time solutions (decrease r_s to decrease D_A)

• increase ρ(z) (extra radiation, EDE) just before ls

→ but need more perturbation

→ worsen S_8 tension, ad hoc matter, coincidence?

$$D_A = \frac{c}{H_0} \int_0^{z_{ls}} \frac{dz}{[\rho(z)/\rho_0]^{1/2}}$$

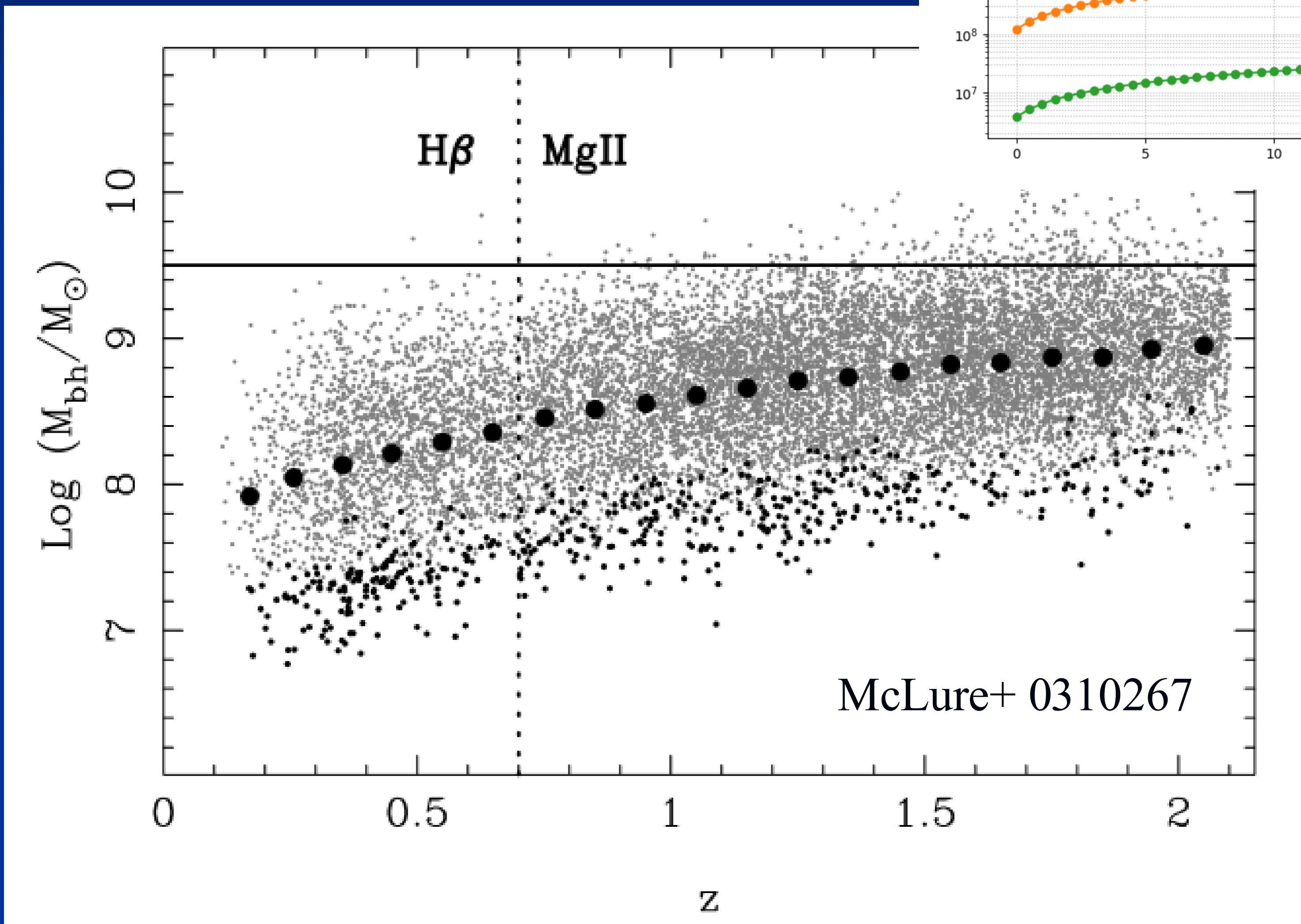
2) in late time solutions (r_s & D_A fixed, increase the integral)

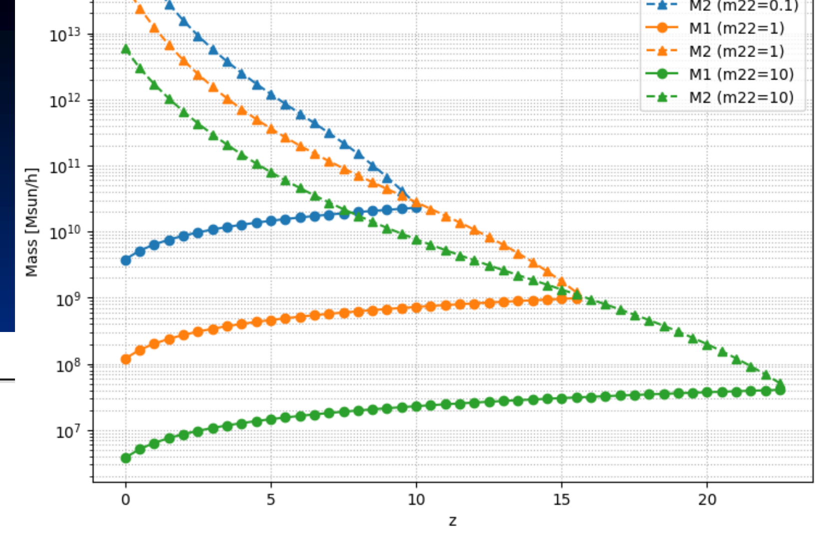
• decrease ρ(z) by decaying DM after ls and/or increase DE to increase the integral

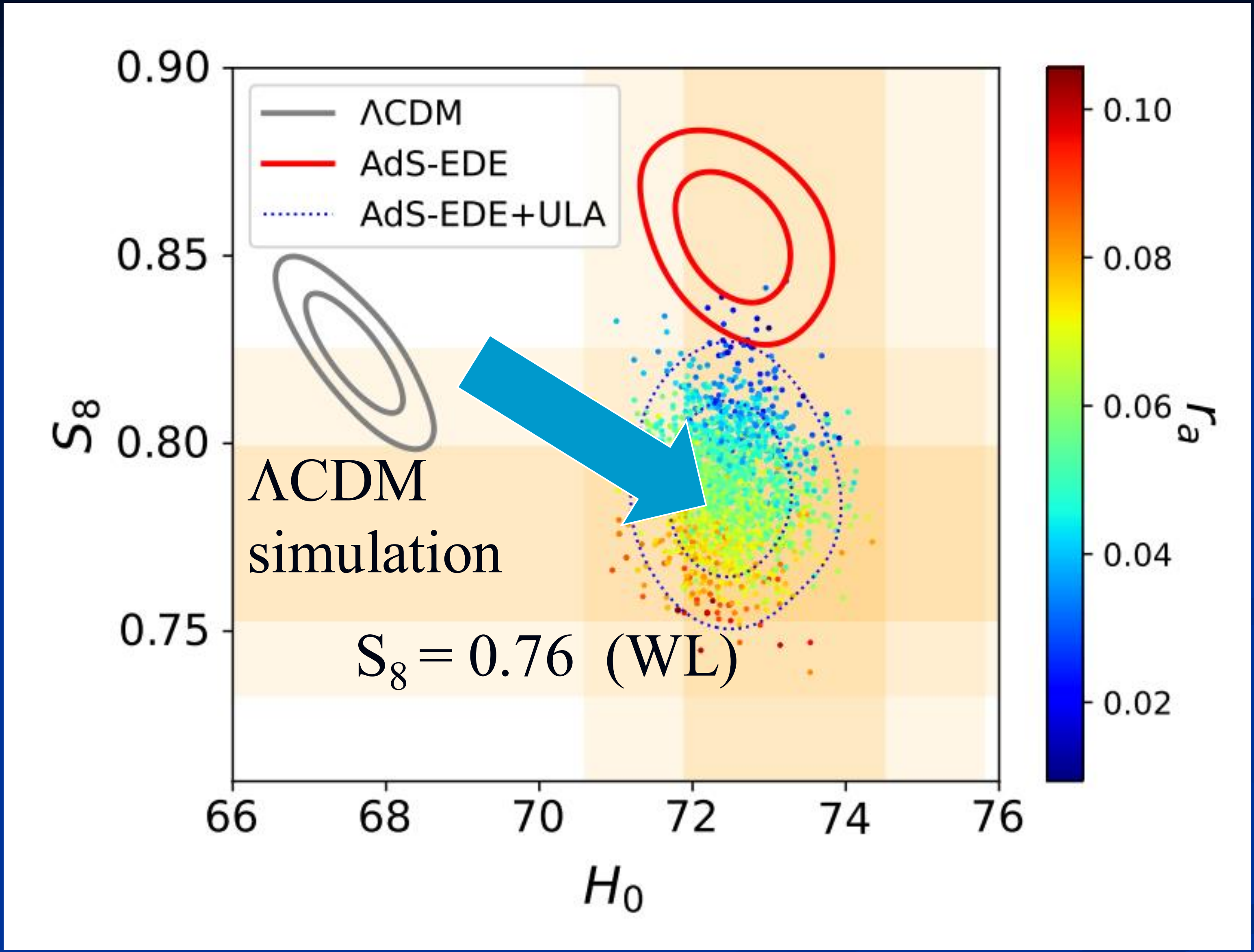
→ late evolution changes

→ tensions with BAO, SN; null energy violation; fine tuning

Do we need both solutions?


Hβ
MgII
Log (M_bh/M_⊙)
10
9
8
7
0
0.5
1
1.5
2
z
McLure+ 0310267


M2 (m22=0.1)
M1 (m22=1)
M2 (m22=1)
M1 (m22=10)
M2 (m22=10)
Mass [Msun/h]
10^13
10^12
10^11
10^10
10^9
10^8
10^7
0
5
10
15
20
z



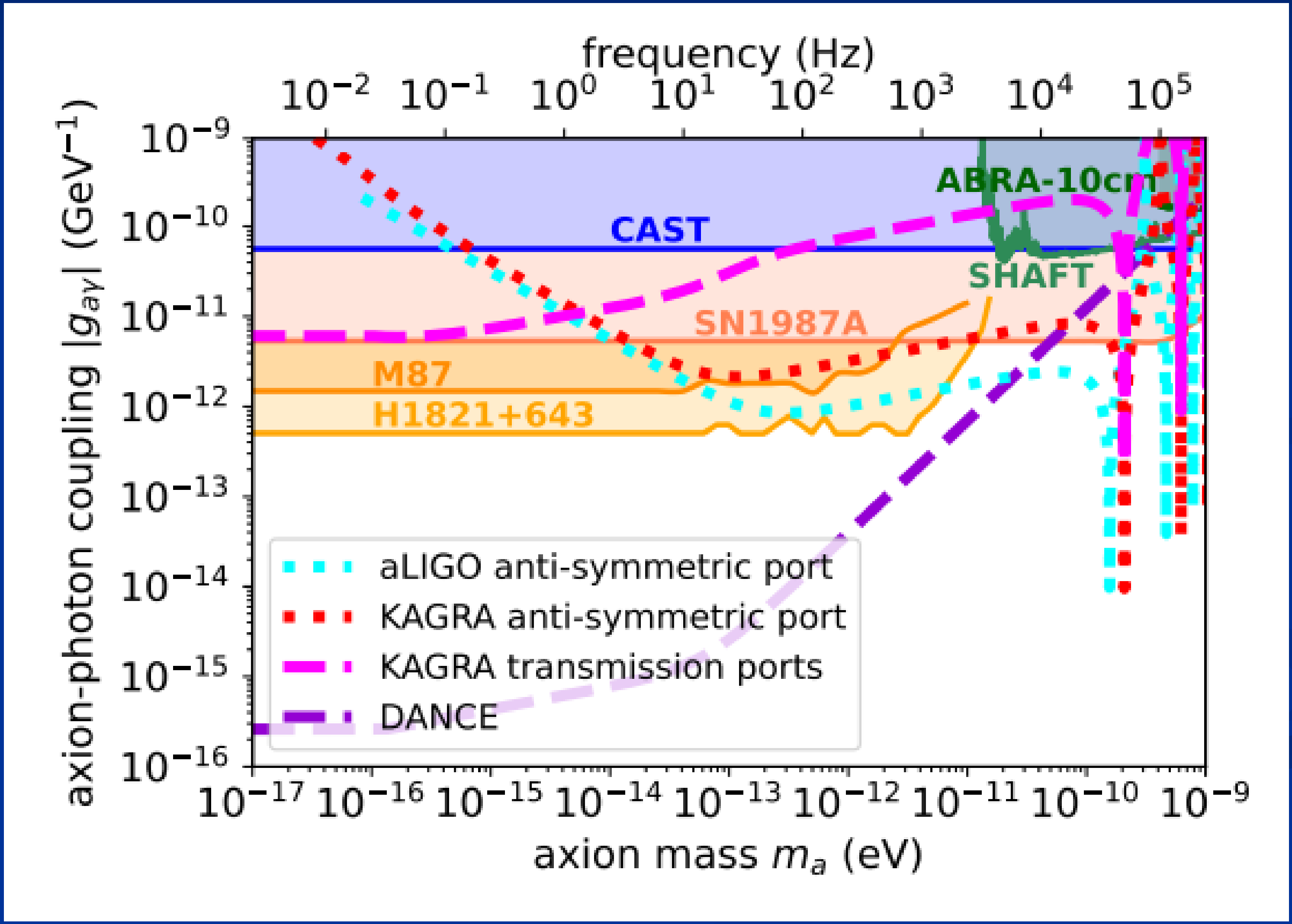
Ye+ 2107.13391

Can ULDM solve both H_0 and S_8 ?

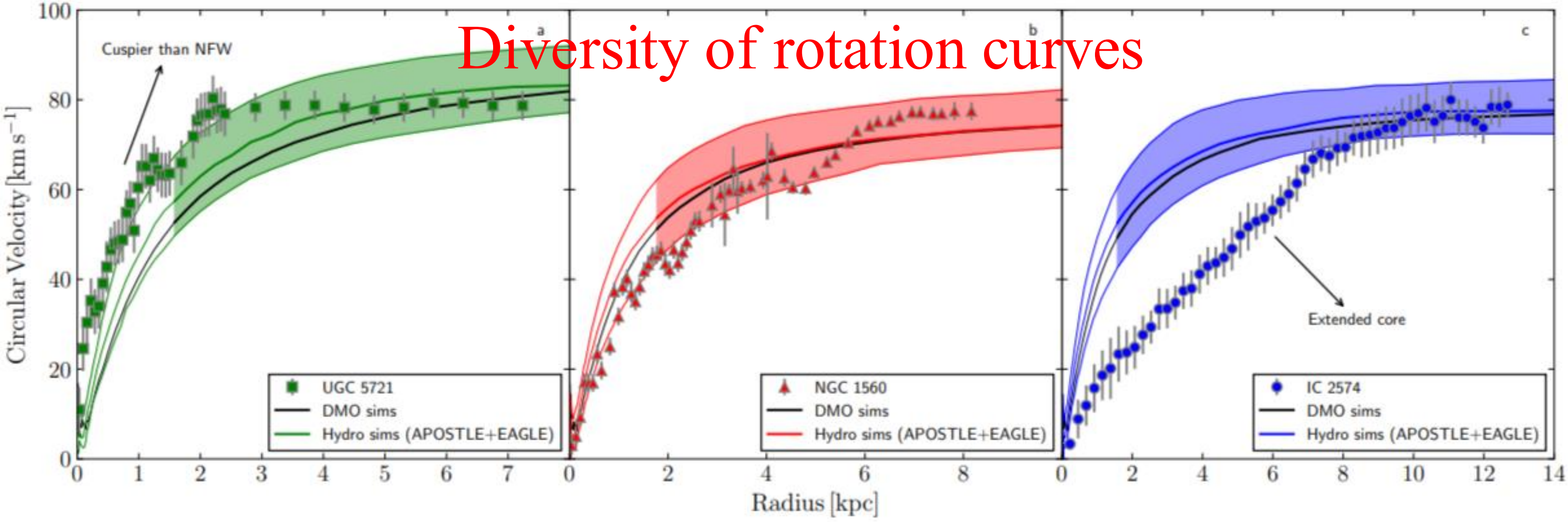
→ need extra DE & ULDM with m < 10^{-24} eV

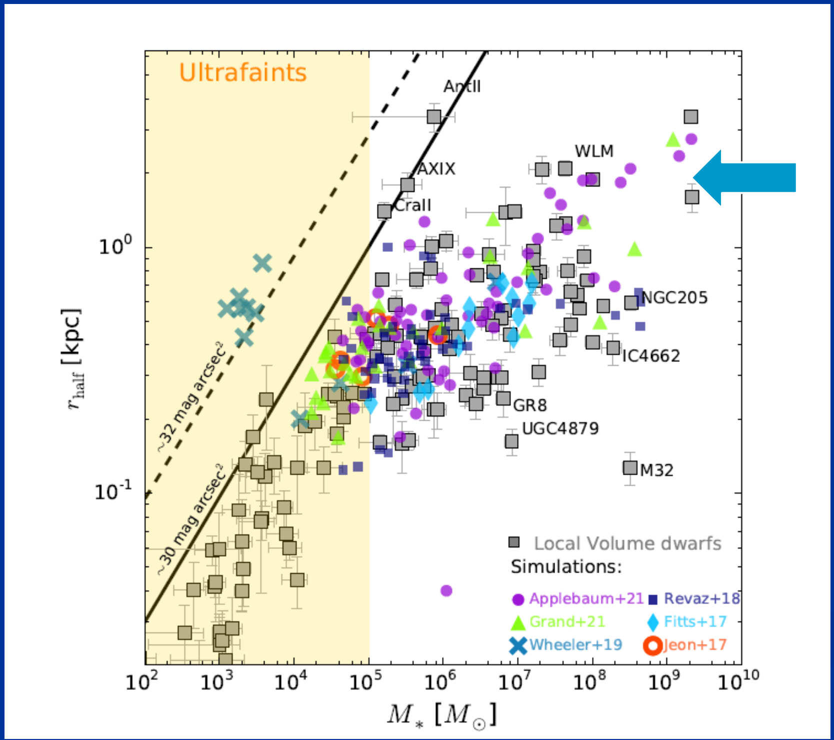
GW

2501.08930



Diversity of rotation curves





Sales+ 2206.05295

a wide range of sizes at fixed stellar mass

No DM models is successful in reproducing RCs of all galaxies so far.

Other cosmological Constraints

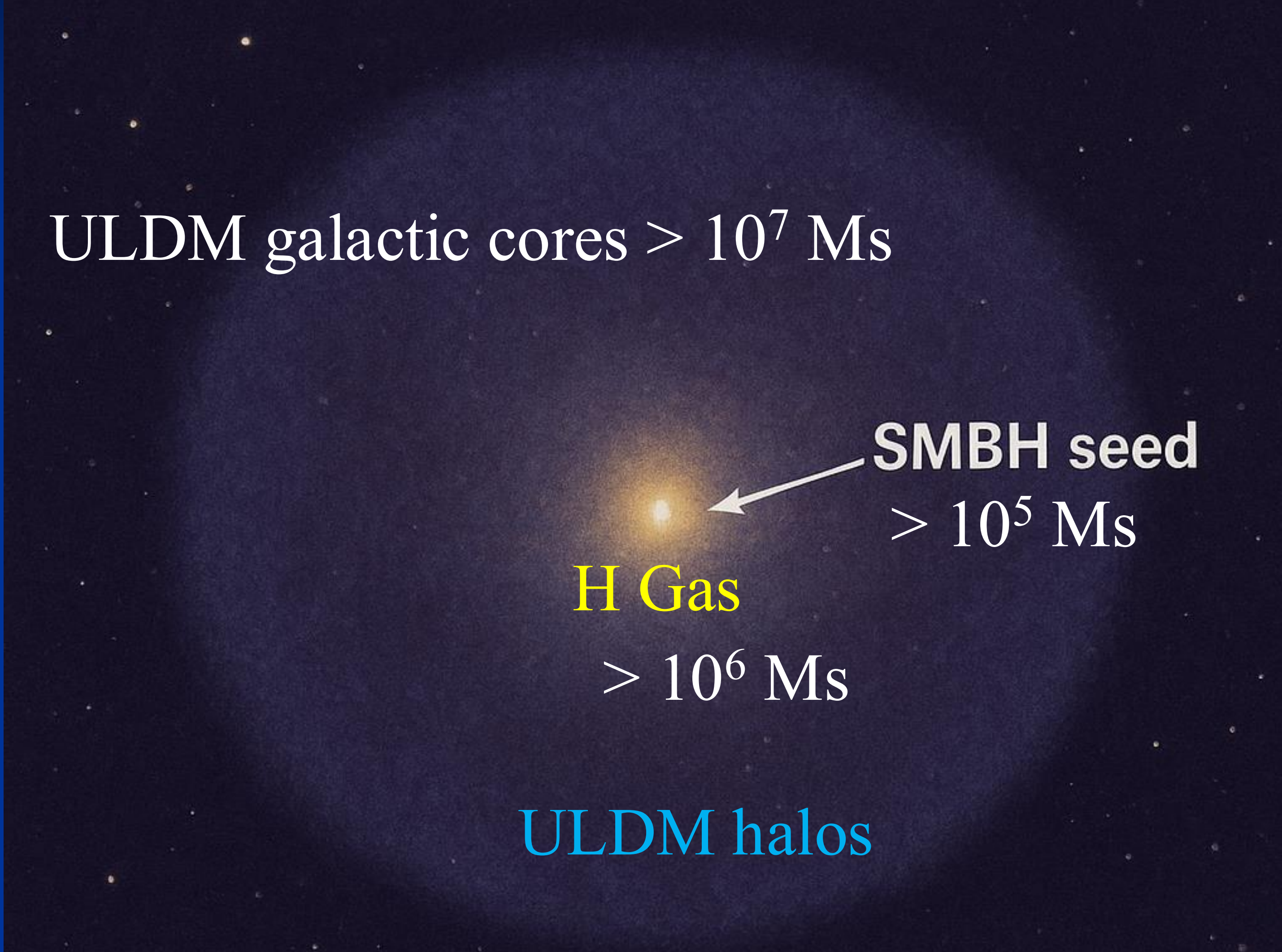
- BEC phase transition before nucleosynthesis: $m < 10^2$ eV
- field oscillation before equality $m > 10^{-28}$eV
- Maximum mass of galaxies from BS theory

  spiral $1.04 \times 10^{12} M_s < O(1) M_p^2/m \rightarrow m < O(1)\ 1.28 \times 10^{-22}$eV

  elliptical $1 \times 10^{13} M_s < O(1) M_p^2/m \rightarrow m < O(1)\ 1.28 \times 10^{-23}$eV
- Lyα forest $m > 10^{-21}$eV
- high-redshift galaxy luminosity → $m > 1.2 \times 10^{-22}$eV
- Stella subpopulations in Fornax → $m < 1.1 \times 10^{-22}$eV
- Ultra-faint dSphs → $m \sim 3.7\text{-}5.6 \times 10^{-22}$eV

fiducial value $m \sim 10^{-22}$eV

Formation of SMBH seeds in ULDM halo



in collaboration with Dongsu BAK (Seoul city Univ.)